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Review

A review of empirical likelihood methods for time series[☆]

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ABSTRACT

We summarize advances in empirical likelihood (EL) for time series data. The EL formulation for independent data is briefly presented, which can apply for inference in special time series problems, reproducing the Wilks phenomenon of chi-square limits for log-ratio statistics. For more general inference with time series, versions of time domain block-based EL, and its generalizations based on divergence measures, are described along with their distributional properties; some approaches are intended for mixing time processes and others are tailored to time series with a Markovian structure. We also present frequency domain EL methods based on the periodogram. Finally, EL for long-range dependent processes is reviewed as well as recent advantages in EL for high dimensional problems. Some illustrative numerical examples are given along with a summary of open research issues for EL with dependent data.

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1. Introduction

For independent, identically distributed (iid) data, Owen (1988, 1990) introduced Empirical Likelihood (EL) as a general nonparametric methodology for creating likelihood-type inference without specifying a joint distributional model for the data, as typical with parametric likelihood. The main idea is to formulate a non-parametric likelihood (or EL) function for assessing the plausibility of values of a given population parameter. The resulting EL function is built by a process of probability profiling of data and leads to likelihood-ratio statistics for constructing tests and confidence regions, which have some analogous properties to their fully parametric likelihood counterparts (e.g., chi-square limits), but often without explicit assumptions about the data-generating mechanism.

Because EL is often intended to be used nonparametrically, extending the methodology to dependent data can be challenging. At issue, an EL formulation often needs to accommodate the unknown (and potentially complex) dependence structure in such data. There have various attempts to develop EL for dependent data, mostly in time series applications. This manuscript attempts to summarize differing approaches for EL inference with time series. We briefly review EL for iid data in Section 2 along with extensions to high-dimensional data. Section 3 presents EL versions for time series which essentially follow the iid data EL formulation with similar distributional properties, despite data dependence. These are typically tailored to a particular model structure or special inference problem, in a way that serial correlation in a time series is not an issue. However, across many general inference problems with time series, the iid data version of EL will generally *fail* and a valid EL formulation needs to nonparametrically accommodate the underlying dependence structure. Data-blocking is a broad technique for this, and Sections 4–6 summarize and compare different block-based EL approaches (e.g., block EL, tapered block EL, regenerative block EL). Further generalizations of block-based EL are described in Section 7. As an alternative to data-blocking, Section 8 describes a frequency domain EL for time series based on a data-transformation. Section 9 then illustrates different EL for time series with a numerical example. Section 10 outlines EL for long-range dependent processes, and Section 11 provides some concluding remarks and open research problems with EL for dependent data.

This EL review is meant to complement some existing ones. Owen's (2001) book provides an accessible account of many developments of EL, including some for dependent data. Chen and Van Keilegom (2009) review EL methods for regression problems, often for independent data but with some connections to dependent data (Section 3.2 here). Both Kitamura (2006) and Bravo (2007) provide summaries of important features of EL for time series inference, with connections to econometrics. Kitamura's (2006) review outlines generalized versions of EL (based on different discrepancy statistics) and their properties, along with associations between EL and general methods of moments estimation (Hansen, 1982). Much of this discussion is related to iid data, but includes a detailed description of distributional properties of a block EL method (Section 4 here). Bravo's (2007) review also describes important uses of EL with general estimating functions in econometric applications (e.g., parameter and moment condition testing) and extends generalized-discrepancy versions of EL mentioned by Kitamura (2006) to time series (cf. Section 7 here). Because Kitamura (2006) and Bravo (2007) detail EL point estimation and moment testing with general possibly over-identifying estimating functions, we do not consider these EL aspects here; these are time series extensions of EL features available for iid data (Qin and Lawless, 1994) as briefly mentioned in Section 2. Rather, we attempt to consolidate general formulations of EL for time series, expanding upon of Velasco's (2009) nice sketch of EL for dependent data.

2. Empirical likelihood under independence

As mentioned in the Introduction, EL inference about a parameter is based on a non-parametric likelihood function built by probability profiling data. For a prototypical example, suppose $X_1, \ldots, X_n$ are iid $\mathbb{R}^d$-valued random vectors and consider inference about their unknown mean $\mathrm{E}X_1 = \mu_0$. Consider a distribution $F(x) = \sum_{i=1}^n p_i \mathbb{I}(X_i \le x)$, $x \in \mathbb{R}^d$, supported on the data and created by assigning a probability p_i to data value X_i, $i = 1, \ldots, n$, such that $\sum_{i=1}^n p_i = 1$, where $\mathbb{I}(\cdot)$ denotes the indicator function. Given the data, a likelihood function for this distribution would be $L(F) \equiv \prod_{i=1}^n [F(X_i) - F(X_i-)] = \prod_{i=1}^n p_i$, which is maximized when F is the empirical distribution $F_n(x) = \sum_{i=1}^n n^{-1}\mathbb{I}(X_i \le x)$ (i.e., $p_i = n^{-1}$). To judge the plausibility of a hypothesized mean value $\mu \in \mathbb{R}^d$, the EL method uses an EL function defined as

$$\begin{aligned} L_n(\mu) &\equiv \sup\{L(F) : F \text{ has mean } \mu \text{ \& is supported on } X_1, \ldots, X_n\} \\ &= \sup\left\{\prod_{i=1}^n p_i : 0 \le p_1, \ldots, p_n \le 1, \sum_{i=1}^n p_i = 1, \sum_{i=1}^n p_i G_i(\mu) = 0_d\right\}, \end{aligned} \quad (1)$$