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On local slope estimation in partial linear models under Gaussian subordination

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ABSTRACT

We address estimation of trend functions and time dependent slope in a partial linear model when the errors are unknown time-dependent functionals of latent Gaussian processes. Asymptotic results are derived under short-memory and long-memory correlations in the data.

1. Introduction

Consider the continuous index bivariate process $\{x(T), y(T),\ T \in \mathbb{R}_+\}$ observed at discrete time points $T_i = i \in \{1, 2, \dots, n\}$. For notational convenience we write $x(T_i) = x_i$ and $y(T_i) = y_i$. Let $t_i = T_i/n = i/n$ denote rescaled times. Consider the equations

$$y_i = g(t_i) + \beta(t_i) \cdot x_i + u_i \tag{1.1}$$

$$x_i = h(t_i) + v_i \tag{1.2}$$

where β, g and h are unknown continuous functions on $[0, 1]$, $u(T_i) = u_i$ and $v(T_i) = v_i$ are error terms having zero means, u_i having finite second moments and v_i having finite fourth moments, u_i and v_i being independent. The assumption that the bivariate stochastic process $\{x(T), y(T),\ T \in \mathbb{R}_+\}$ is a continuous index bivariate process is further exploited for deriving asymptotic formulas and their approximations (see Sections 2, 3 and Appendix).

Eqs. (1.1) and (1.2) define a partial linear model; see Rice (1986), Speckman (1988) and Beran and Ghosh (1998) among others and references therein; also see Engle et al. (1986) and Wahba (1984). Beran and Ghosh (1998) for instance, investigate this model when the slope parameter is a constant and the errors are stationary long-memory processes. Here, our goal is as follows: Using n pairs of observations (x_i, y_i), $i = 1, 2, \dots, n$, we wish to estimate the function $\beta(t)$, $t \in (0, 1)$. Estimation follows by noting in particular that x is a sum of a stochastic component and an unknown but smooth function h. Regression residuals are then used in a follow-up regression model for estimating the slope. This is further explained in Section 2. In particular, we consider kernel estimation although, the results should generalize appropriately to other curve estimation methods as well.

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As for distributional assumptions on the error terms, let u_i and v_i be Gaussian subordinated. Specifically, they are time-dependent one-dimensional transformations of some latent stationary Gaussian processes. Typically, the type of the transformation is unknown and may be non-linear. Although the background Gaussian processes are stationary, due to the transformation the errors may be non-stationary in the sense that their marginal distributions and in particular their variances may change with time. Moreover, their marginal distributions may be non-Gaussian, assuming arbitrary shapes as functions of time. We consider both short memory and long memory correlations in the latent Gaussian processes and derive asymptotic properties of the nonparametric curve estimates. It should be noted that the results of this paper can also be generalized to the case when the continuous index bivariate process $\{(x(T), y(T)), T \in \mathbb{R}_+\}$ is observed at unevenly spaced time points. For simplicity of presentation, we let our observations be evenly spaced in time. However, the assumption that the $(x(T), y(T))$ process is continuous indexed, is exploited further in deriving the asymptotic properties of the curve estimates. See in particular **A9**, **A10** as well as the asymptotic results and their proofs in Section 3 and Appendix.

In previous work, the constant slope case (i.e. $\beta(t) = \beta$) with stationary short-memory errors is considered in Speckman (1988), whereas the long-memory case is considered among others in Beran and Ghosh (1998) and Aneiros-Pérez et al. (2004); also see Robinson (1988) and González-Manteiga and Aneiros-Pérez (2003) and references therein for further background information. To estimate the constant slope parameter, a 'regression through zero' model is fitted to the regression residuals. In the present context, to estimate the time-dependent slope $\beta(t)$, we use a kernel smoothed version of Speckman (1988). Background information on nonparametric curve estimates under long-memory and related references are in Beran and Feng (2002), Csörgő and Mielniczuk (1995), Ghosh (2001), Giraitis and Koul (1997), Giraitis et al. (2012, chapter 12), Guo and Koul (2007), Hall and Hart (1990), Menéndez et al. (2010, 2013), Robinson (1997) and Robinson and Hidalgo (1997). In case of long-memory in the regression errors, bandwidth selection for estimating a nonparametric trend function from unevenly spaced time series observations that are Gaussian subordinated via a monotone transformation is considered in Menéndez et al. (2013). These authors consider only the case of long-range dependence and moreover they do not address uniform convergence of the trend estimate. Under monotonicity, the Hermite rank is unity and the function is invertible. This fact is used by Menéndez et al. (2013) for estimating the latent Gaussian process via estimation of the underlying marginal distribution function of the errors. For discrete time processes and with evenly spaced time series observations, Ghosh and Draghicescu (2002a) consider direct estimation of the variance of the Priestley–Chao estimator. Also for this case, however when the marginal distributions are stationary, Ray and Tsay (1997) and Beran and Feng (2002) propose bandwidth selection methods; also see references therein. For further information on bandwidth selection see Herrmann et al. (1992). As for consistency of the trend estimates, we adopt a simple proof of weak uniform consistency involving the characteristic function of the kernel (Parzen, 1962; Bierens, 1983). The required condition is that the kernel has an absolutely integrable characteristic function. Among other important work on this topic, we draw attention in particular to Hall and Hart (1990), Mack and Silverman (1982), Nadaraya (1965), Schuster (1969) and Silverman (1978); also see references therein. Our partial linear model is in fact a special case of a random design regression model. For related work on this under long-range dependence see among others, Csörgő and Mielniczuk (1999). For background on kernel smoothing see Silverman (1986) and Wand and Jones (1995). Reviews of long-memory processes and their applications in statistics as well as probabilistic backgrounds can be found in Beran (1994), Beran et al. (2013), Giraitis et al. (2012), Leonenko (1999) and Embrechts and Maejima (2002). Our models for the errors is a slight generalization of Taqqu (1975) where however stationarity of the latent Gaussian process is inherited by the subordinated process. Here we let this transformation be time dependent, so as to have the flexibility that the marginal distribution function may change over time. A statistical problem is then the nonparametric prediction of the marginal function at a future time point. This is addressed in Ghosh and Draghicescu (2002b); also see Beran and Ocker (1999). For relevant background information on empirical processes arising from non-linear functionals of Gaussian processes see Breuer and Major (1983), Csörgő and Mielniczuk (1996), Dehling and Taqqu (1989), Dobrushin and Major (1979), Giraitis and Surgailis (1985), Major (1981) and Taqqu (1975, 1979).

The work in this paper is different from the literature in several ways. First of all, the slope parameter is time dependent, so that its local estimation is of interest. Secondly, the errors are assumed to be time dependent non-linear functionals of Gaussian processes, so that their distributions may change over time assuming arbitrary shapes, thus digressing from the often used assumptions of stationarity and Gaussianity. We also address both short-memory and long-memory correlations in the data. A simple proof of uniform consistency of the trend estimates is given extending the line of argument in Parzen (1962) and Bierens (1983).

The paper is organized as follows. Section 2 discusses preliminaries, including technical assumptions (numbered **A1** through **A12**) and terminologies. Slope estimation and related asymptotic results are given in Section 3. Appendix includes the proofs.

2. Preliminaries

Let a_n and b_n be two sequences of real numbers. In what follows, $a_n \sim b_n$ will imply that a_n/b_n converges to a constant as $n \to \infty$. Below, we recollect the model assumptions for (1.1) and (1.2) mentioned above, introduce new notations, terminologies and further assumptions. Thus, for the partial linear model defined in (1.1) and (1.2) we assume that,

- **A1. Trend and slope**. The trend functions $g(t)$ and $h(t)$ as well as the slope function $\beta(t)$ where $t \in [0, 1]$ are in $\mathbb{C}^2[0, 1]$.