Journal of Statistical Planning and Inference 155 (2014) 135–140

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

Minimum breakdown two-level orthogonal main-effect plans

Chen-Tuo Liao [a], Shin-Fu Tsai [b,*]

[a] *Division of Biometry, Institute of Agronomy, National Taiwan University, Taipei 10617, Taiwan*

[b] *Department of Statistics, Feng Chia University, Taichung 40724, Taiwan*

ARTICLE INFO

Article history:
Received 6 August 2013
Received in revised form 15 July 2014
Accepted 16 July 2014
Available online 23 July 2014

Keywords:
Hadamard matrix
Missing value
Orthogonal array
Robust design

ABSTRACT

A new class of robust designs against missing data, called the minimum breakdown designs, is proposed for two-level orthogonal main-effect plans. The minimum breakdown designs provide the highest probability to derive a non-singular residual design for estimating the grand mean and all the main effects, when a fraction of experimental data is unavailable. Some theoretical and computational results for characterizing the minimum breakdown designs are developed. In addition, a catalogue of the proposed designs with practical run-sizes is presented for multifactorial experiments.

1. Introduction

When the number of experimental factors is large, a full factorial design is seldom considered for planning an experiment, primarily due to its uneconomical run-size. Instead, a fractional factorial design is commonly employed for exploring the factor–response relationship. When using a fractional factorial design, the factorial effects of an alias set are inevitably confounded. Typically, the maximum resolution or minimum aberration criterion is used to determine an appropriate design. These two criteria can be thought of as robustness criteria against the aliasing among lower order factorial effects. The reader is referred to Wu and Hamada (2009) for more detailed discussions on the justifications of these criteria. Another issue of design robustness is to take missing observations into consideration. When some observations are missing, the resulting incomplete data usually lead to a loss in estimation efficiency, or in the most extreme situation, the factorial effects of interest are no longer estimable. For example, if missing data are due to certain debarred design points, Cheng and Li (1993) discussed the selection of two-level fractional factorial designs for avoiding the debarred treatment combinations.

To the best of our knowledge, Ghosh (1979) first began to study the robust designs against missing data. Based on his perspective, a design is said to be robust against a loss of i observations, if all the residual designs remain non-singular for estimating all the factorial effects of interest, when any i observations are removed from the original design. Note that a design is said to be non-singular, if the corresponding information matrix is non-singular. This ensures that the factorial effects of interest are all estimable. This line of research was further investigated by John (1979), Ghosh (1980), Moore (1988), Dey (1993), MacEachern et al. (1995), among others. When ranking designs with respect to the robustness presented in Ghosh (1979), there could be several candidate designs robust against a loss of i observations. As a result, a new criterion is required for further discriminating those designs of equal performance. Recently, Tsai and Liao (2013) proposed the minimum breakdown criterion for addressing this design issue. They presented a series of minimum breakdown designs in blocks of size two for the experiment of a single factor. According to the minimum breakdown criterion, robust two-level

* Corresponding author.
E-mail address: sftsai@fcu.edu.tw (S.-F. Tsai).

http://dx.doi.org/10.1016/j.jspi.2014.07.007
0378-3758/

orthogonal main-effect plans (OMEPs) are explored in this study, and designs with economical run-sizes are constructed for practical applications.

The rest of this article is organized as follows. Section 2 introduces the minimum breakdown criterion for quantifying the design robustness against missing data. The design robustness on two-level OMEPs is then explored in Section 3. In addition, a catalogue of robust designs is provided for practical applications. Concluding remarks are given in the final section.

2. Minimum breakdown criterion

Suppose that n two-level factors are considered in an experiment. Let $\boldsymbol{t} = [t_1, t_2, \ldots, t_n]$ be a row vector representing a treatment combination. Note that each entry of $\boldsymbol{t}$ is equal to -1 or 1, which stands for the low or high level of an experimental factor. It is assumed that the observed response can be characterized by the following main-effect model.

$$Y(\boldsymbol{t}) = \mu + \sum_{k=1}^{n} t_k \tau_k + \epsilon(\boldsymbol{t}), \tag{1}$$

where $Y(\boldsymbol{t})$ denotes the response of treatment combination $\boldsymbol{t}$; μ stands for the grand mean; τ_k denotes the main effect of factor k; and $\epsilon(\boldsymbol{t})$ represents the error term. Based on the linear model presented as (1), the N responses can be expressed as the following matrix form.

$$\boldsymbol{Y} = \mu \mathbf{1}_N + \boldsymbol{X}\tau + \epsilon,$$

where $\boldsymbol{Y}$ stands for the vector of responses; $\mathbf{1}_N$ represents the $N \times 1$ vector of ones; and $\boldsymbol{X}$ denotes the $N \times n$ design matrix, in which each row corresponds to a treatment combination, and each column to a factor; τ stands for the vector of main effects; and ϵ represents the vector of error terms, which are assumed to be mutually uncorrelated with a common mean 0 and variance σ^2. As mentioned earlier, a design is said to be non-singular, if the information matrix

$$\begin{bmatrix} N & \mathbf{1}_N^T \boldsymbol{X} \\ \boldsymbol{X}^T \mathbf{1}_N & \boldsymbol{X}^T \boldsymbol{X} \end{bmatrix}$$

is non-singular. The following example is given to demonstrate the main idea of this study.

Example 1. Suppose that a 16-run two-level OMEP is required for estimating the grand mean and the main effects of 8 factors. A simple approach for determining the treatment combinations is to choose 8 columns from a semi-normalized Hadamard matrix of order 16. Note that a Hadamard matrix is said to be semi-normalized, if its first row or first column is a vector of ones. If both of them are vectors of ones, then it is called a normalized Hadamard matrix. Consider a normalized Hadamard matrix Had.16.2 at the website of N.J.A. Sloane (http://neilsloane.com/hadamard/index.html), three designs d_1, d_2 and d_3 consisting of 8 columns of Had.16.2 are given as follows.

$$d_1 = \{2, 3, 4, 5, 6, 7, 9, 16\},$$
$$d_2 = \{2, 3, 4, 5, 6, 9, 10, 11\},$$
$$d_3 = \{2, 3, 4, 5, 6, 7, 8, 9\},$$

where the numbers in the brackets indicate the selected columns. Based on an exhaustive enumeration of all possible residual designs, two residual designs of d_3 are singular for estimating μ and all the main effects, if any two observations are removed. In other words, d_3 is robust against a loss of one observation. On the other hand, all the residual designs of d_1 and d_2 are non-singular, if any three observations are deleted from each design. Namely, both d_1 and d_2 are robust against a loss of three observations. Clearly, d_1 and d_2 are more robust than d_3. However, d_1 and d_2 are of equal performance, and a further discrimination is needed. It can be found that 4 and 40 residual designs of d_1 and d_2 are singular, respectively, if any four observations are removed from each design. Under the assumption that each treatment combination has an equal chance to be missing, d_1 appears to be more robust than d_2, since d_1 provides a higher probability of resulting in a non-singular residual design.

A criterion of refining the robustness proposed by Ghosh (1979) is introduced as follows.

Definition 1. Let d be an N-run main-effect plan for n factors, and M_i represent the number of residual designs which are singular for μ and all the main effects among the $\binom{N}{i}$ possibilities, when any set of i observations is deleted from d. The singular pattern of d is defined as $\boldsymbol{M}(d) = (M_1, M_2, \ldots, M_{N-n-1})$, in which $M_1 \leq M_2 \leq \cdots \leq M_{N-n-1}$.

Based on the notion of design robustness presented in Ghosh (1979), a design d is said to be robust against a loss of i observations, if $M_1 = M_2 = \cdots = M_i = 0$. Under the assumption that each treatment combination has an equal chance to be missing, the probability of resulting in a non-singular residual design is given by

$$P_i = 1 - \frac{M_i}{\binom{N}{i}},$$