Available online at www.sciencedirect.com
ScienceDirect
Journal of Statistical Planning and Inference 137 (2007) 2802–2814
journal of statistical planning and inference
www.elsevier.com/locate/jspi

Improved model selection criteria for SETAR time series models

Pedro Galeano[a,*], Daniel Peña[b]

[a] *Departamento de Estadística e Investigación Operativa, Universidad de Santiago de Compostela, 15782 Santiago de Compostela, Spain*
[b] *Departamento de Estadística, Universidad Carlos III de Madrid, 28903 Getafe, Madrid, Spain*

Received 20 February 2006; received in revised form 18 September 2006; accepted 6 October 2006
Available online 11 February 2007

Abstract

The purpose of this paper is threefold. First, we obtain the asymptotic properties of the modified model selection criteria proposed by Hurvich et al. (1990. Improved estimators of Kullback-Leibler information for autoregressive model selection in small samples. Biometrika 77, 709–719) for autoregressive models. Second, we provide some highlights on the better performance of this modified criteria. Third, we extend the modification introduced by these authors to model selection criteria commonly used in the class of self-exciting threshold autoregressive (SETAR) time series models. We show the improvements of the modified criteria in their finite sample performance. In particular, for small and medium sample size the frequency of selecting the true model improves for the consistent criteria and the root mean square error (RMSE) of prediction improves for the efficient criteria. These results are illustrated via simulation with SETAR models in which we assume that the threshold and the parameters are unknown.

Keywords: Asymptotic efficiency; Autoregressive models; Consistency; Model selection criteria; SETAR models

1. Introduction

Since the seminal work by Akaike (1969), model selection criteria have become a widely used tool for selecting the order of different time series models. Most of these criteria can be classified into two groups. The first one includes the efficient criteria, which asymptotically select the model which produces the least mean square prediction error. The final prediction error criterion (FPE), by Akaike (1969), the Akaike information criterion (AIC), by Akaike (1973) and the corrected Akaike information criterion (AICc), by Hurvich and Tsai (1989) are efficient criteria. The FPE selects the model that minimizes the one step ahead square prediction error. The AIC is an estimator of the expected Kullback–Leibler divergence between the true and the fitted model, while the AICc is a bias correction form of the AIC that appears to work better in small samples. The second group includes the consistent criteria, which, under the assumption that the data come from a finite order autoregressive process, asymptotically select the true order of the process. The Bayesian information criterion (BIC), by Schwarz (1978), and the Hannan–Quinn criterion (HQC), by Hannan and Quinn (1979), are consistent criteria. The BIC approaches the posterior probabilities of the models, while the HQC is designed to be a consistent criterion with the fastest convergence rate to the true model.

* Corresponding author. Tel.: +34 981 563 100x13207; fax: +34 981 597 054.
E-mail address: pgaleano@usc.es (P. Galeano).

0378-3758/$ - see front matter
doi:10.1016/j.jspi.2006.10.014

All these criteria can be written compactly as members of the family of criteria:

$$\min_p \{T \log \widehat{\sigma}_p^2 + (p+1) \times C(T, p+1)\}, \tag{1}$$

where p is the order of the autoregressive process, $\widehat{\sigma}_p^2$ is the maximum likelihood estimate of the residual variance of the process, T is the sample size and $C(T, p+1) = \frac{T}{p+1} \log(\frac{T+p+1}{T-(p+1)})$ for the FPE, $C(T, p+1) = 2$ for the AIC, $C(T, p+1) = \frac{2T}{T-(p+1)-1}$ for the AICc, $C(T, p+1) = \log(T)$, for the BIC and $C(T, p+1) = 2m \log \log(T)$ with $m > 1$, for the HQC.

Hurvich et al. (1990) further approximated the expected Kullback–Leibler divergence to derive a criterion, AICc*, which can be written as follows:

$$\min_p \left\{ \log |\Sigma(\widehat{\alpha}_p)| + \frac{2T(p+1)}{T-(p+1)-1} \right\}, \tag{2}$$

where $|\Sigma(\widehat{\alpha}_p)|$ is the determinant of the estimated covariance matrix of the series under the autoregressive process with order p and parameters α_p, that will be defined in Section 2. These authors also introduced the AIC* and BIC* criteria, by replacing $T \log \widehat{\sigma}_p^2$ by the determinant term in the AIC and BIC criteria, and showed in a Monte Carlo experiment the good performance on this modification. However, they did not study the asymptotic properties of these modified criteria. The first contribution of this paper is to show that the asymptotic properties of the original criteria (1) applies to the modified criteria (2). Thus, we show the efficiency of AIC* and AICc* and the consistency of BIC*.

Although Hurvich et al. (1990) showed via simulation the better performance of the modified criteria, no theoretical reasons have been given explaining this improvements. The second contribution of this paper is to provide three interpretations on the advantages of using the determinant term by using three different comparisons: (1) the one step ahead prediction variances; (2) the correlation structure; (3) a measure of the goodness of the fit.

A useful nonlinear extension of linear time series models are the self-exciting threshold autoregressive (SETAR) models, see Tong (1990). These models can explain interesting features found in real data, such as asymmetric limit cycles, jump phenomena, chaos and so on. Model selection for SETAR models has been addressed in several papers. Tong (1990) suggested to use the AIC but no theoretical justification was given. Wong and Li (1998) showed that the AICc criterion is an asymptotically unbiased estimator of the expected Kullback–Leibler information for SETAR models and analyzed the small sample properties of AIC, AICc and BIC via simulation experiments. Kapetanios (2001) extended some of the existing theoretical results for several model selection criteria in linear models to threshold models. De Gooijer (2001) proposed three cross-validation criteria. Campbell (2004) and Unnikrishnan (2004) developed Bayesian model selection within a Markov Chain Monte Carlo (MCMC) framework, and, finally, Öhrvik and Schoier (2005) studied the performance of several bootstrap selection criteria.

As a SETAR model is piecewise autoregressive linear, it seems natural to extend the modification considered by Hurvich et al. (1990) for autoregressive models to these nonlinear models. The third contribution of this paper is to present new SETAR model selection criteria based on the determinant term and show via a Monte Carlo study the better performance for small and medium sample size of these modified criteria.

The rest of this paper is organized as follows. Section 2 briefly reviews model selection criteria for the class of linear autoregressive models, proves that the correction by the determinant term keeps their asymptotic properties, and provides some intuition to justify why this correction can improve their performance. Section 3 develops the modification by the determinant term for SETAR time series model selection criteria. Section 4 shows the better performance of the modified criteria in a Monte Carlo experiment.

2. Model selection criteria for the class of linear autoregressive processes

2.1. Model selection criteria for autoregressive processes

Suppose it is known that a given time series, $x = (x_1, \ldots, x_T)'$, has been generated by the class of autoregressive (AR) Gaussian processes, given by

$$x_t - \phi_1 x_{t-1} - \cdots - \phi_p x_{t-p} = a_t, \quad t = \ldots, -1, 0, 1, \ldots,$$