

A goodness-of-fit process for ARMA(p, q) models based on a modified residual autocorrelation sequence

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Abstract

The asymptotic distribution of the usual goodness-of-fit process for ARMA(p, q) models based on the residual autocorrelations is obtained. This distribution depends on unknown parameters. This fact motivates the introduction of a modified goodness-of-fit process based on a transformation of the residual autocorrelations. This new process is shown to converge weakly to the Brownian bridge. Thus, functionals based on the modified process are adequate for goodness-of-fit purposes. The behavior of these functionals is analyzed and compared to that of the standard Ljung–Box and cumulative periodogram statistics. The new method is found to perform better in some simulation experiments at the cost of some increased numerical complexity.

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1. Introduction

Consider a causal and invertible ARMA(p, q) process with mean μ :

$$\phi(B)(X_t - \mu) = \theta(B)\varepsilon_t, \quad (1)$$

where B is the backward shift operator $BX_t = X_{t-1}$ and $\{\varepsilon_t : t \in \mathbb{Z}\}$ is a zero mean white noise sequence with variance σ^2 . Also, $\phi(z) = 1 - \phi_1 z - \dots - \phi_p z^p$ and $\theta(z) = 1 + \theta_1 z + \dots + \theta_q z^q$ are polynomials of degrees p and q , respectively. It will be convenient to define $P = \max(p, q)$.

Given a finite observed series $(X_1, \dots, X_n)'$, the mean μ can be estimated by the average $\bar{X}_n = \sum_{i=1}^n X_i / n$. The parameters $\phi = (\phi_1, \dots, \phi_p)'$ and $\theta = (\theta_1, \dots, \theta_q)'$ can be estimated by the least squares estimates

$$(\hat{\phi}, \hat{\theta}) = \arg \min_{(\phi, \theta)} \sum_{t=P}^n [\varepsilon_t(\phi, \theta, \bar{X}_n)]^2, \quad (2)$$

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where the functions $\{\varepsilon_t(\boldsymbol{\phi}, \boldsymbol{\theta}, \mu) : 1 \leq t \leq n\}$ are defined implicitly by means of equations $\phi(B)(X_t - \mu) = \theta(B)\varepsilon_t(\boldsymbol{\phi}, \boldsymbol{\theta}, \mu)$ and conditions $X_t - \mu \equiv 0 \equiv \varepsilon_t(\boldsymbol{\phi}, \boldsymbol{\theta}, \mu)$, $t \leq 0$. Under adequate assumptions, $\sqrt{n}[(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\theta}}) - (\boldsymbol{\phi}, \boldsymbol{\theta})] \xrightarrow{D} N_{p+q}[0, \mathbf{I}^{-1}(\boldsymbol{\phi}, \boldsymbol{\theta})]$, where $\mathbf{I}(\boldsymbol{\phi}, \boldsymbol{\theta})$ is the $(p+q) \times (p+q)$ information matrix for the parameters $(\boldsymbol{\phi}, \boldsymbol{\theta})$ (see e.g. Brockwell and Davis, 1991, Chapter 8). The least squares residuals are $\hat{\varepsilon}_t = \varepsilon_t(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\theta}}, \bar{X}_n)$. Then, for $P < t \leq n$, $\hat{\varepsilon}_t = (X_t - \bar{X}_n) - \sum_{j=1}^P \hat{\phi}_j(X_{t-j} - \bar{X}_n) - \sum_{j=1}^q \hat{\theta}_j \hat{\varepsilon}_{t-j}$. The residual autocovariances are $\hat{g}_k = \sum_{t=P+k}^n \hat{\varepsilon}_t \hat{\varepsilon}_{t+k} / n$, $0 \leq k \leq n - (P+1)$. Hence, the residual autocorrelations are $\hat{r}_k = \hat{g}_k / \hat{g}_0$, $1 \leq k \leq n - (P+1)$. The latter are useful for investigating the adequacy of the fit to $(X_1, \dots, X_n)'$ of an ARMA(p, q) model of the form (1). A standard goodness-of-fit method is the Ljung and Box (1978) statistic

$$\hat{Q}_n^m = n(n+2) \sum_{k=1}^m (n-k)^{-1} \hat{r}_k^2, \quad (3)$$

where $m = m_n$ is a function of the sample size n . Under model (1), the approximate distribution for n large of (3) is chi squared with $m - (p+q)$ degrees of freedom. For a review of the properties of $\hat{Q}_n^m$ and other diagnostic checks in time series based on the residual autocorrelations see Li (2004, Chapter 2).

The residual autocorrelations can be also used to test for goodness-of-fit in the frequency domain. Define the standardized residual *periodogram* ordinates

$$\hat{I}_n(\lambda) = \frac{1}{2\pi n \hat{g}_0} \left| \sum_{t>P} \hat{\varepsilon}_t \exp(-i\lambda t) \right|^2 = \frac{1}{2\pi} \left[1 + 2 \sum_{k=1}^{n-(P+1)} \hat{r}_k \cos(k\lambda) \right], \quad (4)$$

where $0 \leq \lambda \leq \pi$. Define also the standardized residual spectral distribution function $\hat{F}_n(\lambda) = 2 \int_0^\lambda \hat{I}_n(t) dt = [\lambda + 2 \sum_{k=1}^{n-(P+1)} \hat{r}_k \sin(k\lambda)/k] / \pi$. Put $F_0(\lambda) = 2 \int_0^\lambda f_0(t) dt = \lambda/\pi$, where $f_0(\lambda) = (2\pi)^{-1}$ is the standardized spectral density of a white noise sequence. Consider the stochastic process $\{\hat{W}_n(u) : 0 \leq u \leq 1\}$, where

$$\hat{W}_n(u) = \sqrt{\frac{n}{2}} [\hat{F}_n(\pi u) - F_0(\pi u)] = \frac{\sqrt{2}}{\pi} \sqrt{n} \sum_{k=1}^{n-(P+1)} \hat{r}_k \frac{\sin(k\pi u)}{k} \quad (5)$$

is a random element in $C[0, 1]$, the space of continuous functions in $[0, 1]$.

Inference on the appropriateness of an ARMA(p, q) model could be based on test statistics of the form $H[\hat{W}_n(u)]$, where $H[\cdot]$ is a suitable continuous functional of the goodness-of-fit process (5). This is because, if model (1) is adequate, the residuals $\hat{\varepsilon}_t \cong \varepsilon_t$ should exhibit white noise characteristics. Consequently, $\hat{W}_n(u)$ can be expected to behave similarly as

$$W_n(u) = \frac{\sqrt{2}}{\pi} \sqrt{n} \sum_{k=1}^{n-1} r_k \frac{\sin(k\pi u)}{k}, \quad (6)$$

where the $\{r_k : 1 \leq k \leq n-1\}$ are the sample autocorrelations of the errors $\{\varepsilon_t : 1 \leq t \leq n\}$. These are defined as $r_k = g_k / g_0$, where $g_k = \sum_{t=1}^{n-k} \varepsilon_t \varepsilon_{t+k} / n$, $0 \leq k \leq n-1$. The asymptotic theory of $\{W_n(u) : 0 \leq u \leq 1\}$ has been studied by several authors. See, for example, Bartlett (1966), Grenander and Rosenblatt (1957), Priestley (1981, section 6.2.6), and Dahlhaus (1985). Under adequate assumptions, Durlauf (1991, section 2) and Anderson (1993, section 2) proved that process (6) converges weakly in $C[0, 1]$ to the Brownian bridge $\{B(u) : 0 \leq u \leq 1\}$. From the continuous mapping theorem (Billingsley, 1999, theorem 2.7), the significance of an observed value of $H[\hat{W}_n(u)]$ could be assessed by taking as a reference the distribution of $H[B(u)]$, the corresponding functional of the Brownian bridge. However, Durbin (1975, section 2) argues that $\{\hat{W}_n(u) : 0 \leq u \leq 1\}$ does not converge to $\{B(u) : 0 \leq u \leq 1\}$, but to a zero mean Gaussian process $\{G(u) : 0 \leq u \leq 1\}$ with covariance function depending on the parameters $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$. Thus, the approximation to the null distribution of $H[\hat{W}_n(u)]$ provided by $H[B(u)]$ may not be very reliable in applications.

This paper reviews, in Section 2, the work of Durbin (1975) on process (5). Specific techniques of weak convergence in $C[0, 1]$ are used to give a derivation of the limit behavior of $\{\hat{W}_n(u) : 0 \leq u \leq 1\}$. The method of proof suggests a transformed residual autocorrelation sequence that leads to a modified goodness-of-fit process $\{\hat{Z}_n(u) : 0 \leq u \leq 1\}$. In Section 3, this new process is shown to converge weakly to the Brownian bridge. Thus, inference on the adequacy of

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