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On parameter estimation of stochastic delay differential equations with guaranteed accuracy by noisy observations☆

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Abstract

Let $(X(t), t \geqslant -1)$ and $(Y(t), t \geqslant 0)$ be stochastic processes satisfying

$$\mathrm{d}X(t) = aX(t)\,\mathrm{d}t + bX(t-1)\,\mathrm{d}t + \mathrm{d}W(t)$$

and

$$\mathrm{d}Y(t) = X(t)\,\mathrm{d}t + \mathrm{d}V(t),$$

respectively. Here $(W(t), t \geqslant 0)$ and $(V(t), t \geqslant 0)$ are independent standard Wiener processes and $\vartheta = (a, b)'$ is assumed to be an unknown parameter from some subset Θ of $\mathscr{R}^2$.

The aim here is to estimate the parameter ϑ based on continuous observation of $(Y(t), t \geqslant 0)$.

Sequential estimation plans for ϑ with preassigned mean square accuracy ε are constructed using the so-called correlation method. The limit behaviour of the duration of the estimation procedure is studied if ε tends to zero.

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1. Introduction

Assume $(\Omega, \mathscr{F}, (\mathscr{F}(t), t \geqslant 0), P)$ is a given filtered probability space and the processes $W = (W(t),\ t \geqslant 0)$ and $V = (V(t), t \geqslant 0)$ are real-valued standard Wiener processes on $(\Omega, \mathscr{F}, (\mathscr{F}(t),\ t \geqslant 0), P)$, adapted to $(\mathscr{F}(t))$ and mutually independent. Further assume that $X_0 = (X_0(t), t \in [-1, 0])$ and Y_0 are a real-valued cadlag process and a real-valued random variable, respectively, on $(\Omega, \mathscr{F}, (\mathscr{F}(t), t \geqslant 0), P)$ with

$$E\int_{-1}^{0} X_0^2(s)\,\mathrm{d}s < \infty \quad \text{and} \quad EY_0^2 < \infty.$$

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Assume that Y_0 and $X_0(s)$ are $\mathscr{F}_0$-measurable for every s from $[-1, 0]$ and that the quantities $W,\ V,\ X_0$ and Y_0 are mutually independent.

Consider a two-dimensional random process $(X, Y) = (X(t), Y(t))$ described by the system of stochastic differential equations

$$\mathrm{d}X(t) = aX(t)\,\mathrm{d}t + bX(t-1)\,\mathrm{d}t + \mathrm{d}W(t), \tag{1}$$

$$\mathrm{d}Y(t) = X(t)\,\mathrm{d}t + \mathrm{d}V(t), \quad t \geqslant 0 \tag{2}$$

with the initial conditions $X(t) = X_0(t),\ t \in [-1, 0]$ and $Y(0) = Y_0$. The process X is supposed to be hidden, i.e., unobservable, and the process Y is observed. Such models are used in applied problems connected with control, filtering and prediction of stochastic processes (see, for example, Arato, 1982; Liptser and Shiryaev, 1977).

The parameter $\vartheta = (a, b)'$ with $a, b \in \mathscr{R}^1$ is assumed to be unknown and shall be estimated by using one continuous observation of Y.

Eqs. (1) and (2) together with the initial values $X_0(\cdot)$ and Y_0, respectively, have uniquely solutions $X(\cdot)$ and $Y(\cdot)$, for details see Mao (1997). Eq. (1) is a very special case of stochastic differential equations with time delay, see Kolmanovskii and Myshkis (1992), Mohammed (1996) and Küchler and Sørensen (1997) for examples.

To estimate the true parameter ϑ with a preassigned least square accuracy ε we shall construct sequential plans $(T_\varepsilon, \vartheta_\varepsilon^*)$. Moreover, we will derive asymptotic properties of the duration T_ε of these plans for ε tending to zero.

The method used below is to transform Eqs. (1) and (2) to a single equation (see (6)) for the process $(Y(t), t \geqslant 0)$, which can be treated by modifying a method from Vasiliev and Konev (1987). The construction of $(T_\varepsilon, \vartheta_\varepsilon^*)$ may depend on the asymptotic behaviour of the correlation function of the solution of (1) and its estimators if the observation time is increasing unboundedly. These asymptotic properties vary if ϑ runs through $\mathscr{R}^2$. Our construction does not seem to work for all ϑ in $\mathscr{R}^2$. Therefore we restrict the discussion to two sets Θ_1 and Θ_2 of parameters, for which we are able to derive the desired properties.

The organization of this paper is as follows. In Section 2 we summarize some known properties of Eq. (1) needed in the sequel. The two mentioned cases for Θ, namely Θ_1 and Θ_2, are presented and Eqs. (1) and (2) are transformed into a new one for the one-dimensional observed process $(Y(t), t \geqslant 0)$ (see (6)). In Section 3 the two sequential plans are constructed and the assertions are formulated. Section 4 contains the proofs.

2. Preliminaries

First we summarize some known facts about Eq. (1). For details the reader is referred to Gushchin and Küchler (1999). Together with the described initial condition Eq. (1) has a uniquely determined solution X which can be represented as follows for $t \geqslant 0$:

$$X(t) = x_0(t)X_0(t) + b\int_{-1}^{0} x_0(t-s-1)X_0(s)\,\mathrm{d}s + \int_0^t x_0(t-s)\,\mathrm{d}W(s). \tag{3}$$

Here $x_0 = (x_0(t), t \geqslant -1)$ denotes the so-called fundamental solution of the deterministic equation

$$x_0(t) = 1 + \int_0^t (\vartheta_0 x_0(s) + \vartheta_1 x_0(s-1))\,\mathrm{d}s, \quad t \geqslant 0, \tag{4}$$

corresponding to (1) with $x_0(t) = 0,\ t \in [-1, 0),\ x_0(0) = 1$.

The solution X has the property $E\int_0^T X^2(s)\,\mathrm{d}s < \infty$ for every $T > 0$.

The limit behaviour of $x_0(t)$ and therefore also of $X(t)$ for t tending to infinity is closely connected with the properties of the set $\Lambda = \{\lambda \in \mathbb{C} | \lambda = a + b\mathrm{e}^{-\lambda}\}$($\mathbb{C}$ denotes the set of complex numbers). The set Λ is countable infinite (if $b \neq 0$), and for every real c the set $\Lambda_c = \Lambda \cap \{\lambda \in \mathbb{C} |\ \mathrm{Re}\,\lambda \geqslant c\}$ is finite. In particular, $v_0 := v_0(\vartheta) = \sup\{\mathrm{Re}\,\lambda | \lambda \in \Lambda\} < \infty,\ \sup\{\emptyset\} = -\infty$. Define $v_1(\vartheta) := \sup\{\mathrm{Re}\,\lambda | \lambda \in \Lambda,\ \mathrm{Re}\,\lambda < v_0(\vartheta)\}$.

The values $v_0(\vartheta)$ and $v_1(\vartheta)$ determine the asymptotic behaviour of $x_0(t)$ as $t \to \infty$. Indeed, it exist a real γ less than v_1 and a polynomial $\Psi_1(\cdot)$ of degree less than or equal to one, being specified in the proof of Theorem 3.1 (Section 4), such that

$$x_0(t) = \frac{1}{v_0 - a + 1}\mathrm{e}^{v_0 t} + \Psi_1(t)\mathrm{e}^{v_1 t} + \mathrm{o}(\mathrm{e}^{\gamma t}) \quad \text{as } t \to \infty.$$