Available online at www.sciencedirect.com
ScienceDirect
Journal of Statistical Planning and Inference 137 (2007) 3082–3091
journal of statistical planning and inference
www.elsevier.com/locate/jspi

Factor stochastic volatility with time varying loadings and Markov switching regimes

Hedibert Freitas Lopes[a,*], Carlos Marinho Carvalho[b]

[a]Graduate School of Business, University of Chicago, 5807 South Woodlawn Avenue, Chicago, IL 60637, USA
[b]Institute of Statistics and Decision Sciences, Duke University, 214 Old Chemistry Building, Durham, NC 27708-0251, USA

Received 25 September 2005; accepted 29 June 2006
Available online 15 March 2007

Abstract

We generalize the factor stochastic volatility (FSV) model of Pitt and Shephard [1999. Time varying covariances: a factor stochastic volatility approach (with discussion). In: Bernardo, J.M., Berger, J.O., Dawid, A.P., Smith, A.F.M. (Eds.), Bayesian Statistics, vol. 6, Oxford University Press, London, pp. 547–570.] and Aguilar and West [2000. Bayesian dynamic factor models and variance matrix discounting for portfolio allocation. J. Business Econom. Statist. 18, 338–357.] in two important directions. First, we make the FSV model more flexible and able to capture more general time-varying variance–covariance structures by letting the matrix of factor loadings to be time dependent. Secondly, we entertain FSV models with jumps in the common factors volatilities through So, Lam and Li's [1998. A stochastic volatility model with Markov switching. J. Business Econom. Statist. 16, 244–253.] Markov switching stochastic volatility model. Novel Markov Chain Monte Carlo algorithms are derived for both classes of models. We apply our methodology to two illustrative situations: daily exchange rate returns [Aguilar, O., West, M., 2000. Bayesian dynamic factor models and variance matrix discounting for portfolio allocation. J. Business Econom. Statist. 18, 338–357.] and Latin American stock returns [Lopes, H.F., Migon, H.S., 2002. Comovements and contagion in emergent markets: stock indexes volatilities. In: Gatsonis, C., Kass, R.E., Carriquiry, A.L., Gelman, A., Verdinelli, I., Pauler, D., Higdon, D. (Eds.), Case Studies in Bayesian Statistics, vol. 6, pp. 287–302].

MSC: 62C10; 62P20; 62F40; 91B70; 91B82

Keywords: Bayesian inference; Factor analysis; Variance decomposition; Dynamic models; Markov switching

1. Introduction

A vast literature on Bayesian analysis of univariate stochastic volatility (SV) processes have appeared after the seminal work of Jacquier et al. (JPR 1994) who perform fully Bayesian inference through a Markov chain Monte Carlo (MCMC) scheme. For instance, Kim et al. (1998) replace JPR's scheme by a forward filtering–backward sampling (FFBS) step (Carter and Kohn, 1994; Frühwirth-Schnatter, 1994) when sampling the log-volatilities. Jensen (2004) develops semiparametric inference for long-memory SV processes, while So et al. (1998) and Carvalho and Lopes (2007) accommodate Markov jumps in the log-volatilities.

* Corresponding author. Tel.: +1 773 834 5458; fax: +1 773 702 0458.
E-mail address: hlopes@ChicagoGSB.edu (H.F. Lopes).

0378-3758/$ - see front matter
doi:10.1016/j.jspi.2006.06.047

The literature on multivariate SV models is less abundant, with the works of Harvey et al. (1994), Pitt and Shephard (1999), Aguilar and West (2000), Lopes and Migon (2002) and Chib et al. (2006) forming the basis for the developments we consider here. Roughly speaking, they model the levels (or first differences) of a set of (financial) time-series by a standard normal factor model (Lopes and West, 2004) in which both the common factor variances and the specific (or idiosyncratic) time-series variances are modeled as univariate SV processes. The main practical and computational advantage of the factor stochastic volatility (FSV) model is its parsimony, where all the variances and covariances of a vector of time-series are modeled by a low-dimensional SV structure dictated by common factors. It is fairly common to find that, for large vectors of time-series, the number of common factors is usually one or two orders of magnitude smaller, which speeds up computation and estimation considerably.

Our main contribution is twofold. First, we let the factor loading matrix to be time-dependent allowing the FSV model to capture more general time-varying correlation structures. Second, we extend So et al. (1998) and Carvalho and Lopes (2007) Markov jumps to model the common factors' stochastic log-volatilities. We start, in Section 2, with a general introduction to FSV models followed by the description of the proposed extensions in Section 3. Bayesian inference and computation are developed in Section 4 where customized MCMC algorithms are presented. The extensions are illustrated through two financial time-series examples in Section 5 followed by a final section of discussions and thoughts on directions for future investigation.

2. FSV models: a brief review

In this section we briefly review standard factor analysis and FSV models, as well as modeling and identification issues.

2.1. Standard factor analysis

Standard normal factor analysis is the backbone of FSV models. In this context, data on p related variables are considered to arise through random sampling from a zero-mean multivariate normal distribution where $\mathbf{\Omega}$ denotes a $p \times p$ non-singular variance matrix. For any specified positive integer $q \leqslant p$, the standard q-factor model relates each $\mathbf{y}_t$ to an underlying q-vector of random variables $\boldsymbol{f}_t$, the common factors, via

$$\mathbf{y}_t = \boldsymbol{\beta}\boldsymbol{f}_t + \epsilon_t, \tag{1}$$

where (i) the factors $\boldsymbol{f}_t$ are independent with $\boldsymbol{f}_t \sim \mathrm{N}(\mathbf{0}, \mathbf{I}_q)$, (ii) the ϵ_t are independent normal p-vectors with $\epsilon_t \sim \mathrm{N}(\mathbf{0}, \mathbf{\Sigma})$, and $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_p^2)$, (iii) ϵ_t and $\boldsymbol{f}_s$ are independent for all t and s and (iv) $\boldsymbol{\beta}$ is the $p \times q$ factor loadings matrix.

Under this model, the variance–covariance structure of the data distribution is constrained with $\mathbf{\Omega} = V(\mathbf{y}_t|\mathbf{\Omega}) = V(\mathbf{y}_t|\boldsymbol{\beta}, \mathbf{\Sigma}) = \mathbf{\Omega} = \boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{\Sigma}$. Conditional on the common factors, observable variables are uncorrelated. In other words, the common factors explain all the dependence structures among the p variables. For any element y_{it} and y_{jt} of $\mathbf{y}_t$ and conditionally on $\boldsymbol{\beta}$ and $\mathbf{\Sigma}$, we have the characterizing moments, (i) $\mathrm{var}(y_{it}|\boldsymbol{f}) = \sigma_i^2$, (ii) $\mathrm{cov}(y_{it}, y_{jt}|\boldsymbol{f}) = 0$, (iii) $\mathrm{var}(y_{it}) = \sum_{l=1}^q \beta_{il}^2 + \sigma_i^2$ and (iv) $\mathrm{cov}(y_{it}, y_{jt}) = \sum_{l=1}^q \beta_{il}\beta_{jl}$.

In practical problems, especially with larger values of p, the number of factors q will often be small relative to p, so most of the variance–covariance structure is explained by a small number of common factors. The *uniquenesses*, or *idiosyncratic variances*, σ_i^2 measure the residual variability in each of the data variables once that contributed by the factors is accounted for. Modern MCMC-based posterior inference in standard factor analysis appears in, among others, Geweke and Zhou (1996) and Lopes and West (2004).

2.2. Model structure and identification issues

The q-factor model is invariant under transformations of the form $\boldsymbol{\beta}^* = \boldsymbol{\beta}\boldsymbol{P}'$ and $\boldsymbol{f}_t^* = \boldsymbol{P}\boldsymbol{f}_t$, where $\boldsymbol{P}$ is any orthogonal $q \times q$ matrix. There are many ways of identifying the model by imposing constraints on $\boldsymbol{\beta}$, including constraints to orthogonal $\boldsymbol{\beta}$ matrices, and constraints such that $\boldsymbol{\beta}'\mathbf{\Sigma}^{-1}\boldsymbol{\beta}$ is diagonal. The alternative preferred here is to constrain $\boldsymbol{\beta}$ so that it is a block lower triangular matrix, assumed to be of full rank, with diagonal elements strictly positive. This form is used, for example, in Geweke and Zhou (1996), Aguilar and West (2000) and Lopes and Migon (2002), and provides