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Rényi statistics for testing hypotheses in mixed linear regression models ☆

I. Molina[a,*], D. Morales[b]

[a] Universidad Carlos III de Madrid, C/Madrid, 126 - 28903 Getafe, Madrid, Spain
[b] Universidad Miguel Hernández de Elche, Avenida de la Universidad, s/n - 03202 Elche, Spain

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Abstract

Rényi divergences are used to propose some statistics for testing general hypotheses in mixed linear regression models. The asymptotic distribution of these tests statistics, of the Kullback–Leibler and of the likelihood ratio statistics are provided, assuming that the sample size and the number of levels of the random factors tend to infinity. A simulation study is carried out to analyze and compare the behavior of the proposed tests when the sample size and number of levels are small.

MSC: 62B10; 62E20

Keywords: Rényi divergences; Kullback–Leibler test; Mixed linear models; Random effects

1. Introduction to mixed linear regression models

Random effects models, and more generally mixed linear models, have been studied over many years in the research literature. Some books devoted to the study of these models are e.g. Rao and Kleffe (1988), Searle et al. (1992), Rao (1997) and, more recently, Sahai and Ojeda (2004). Applications can be found in many fields, like Quantitative Genetic Theory (Fisher, 1918), Agricultural Sciences (Yates and Zacopanay, 1935), Psychology ((Jackson, 1939), Marine Biology (Winsor and Clarke, 1940), Animal Sciences (Henderson, 1969), and more recently Small Area Estimation (Rao, 2003). However, so far the main interest has been put on estimation, while hypotheses testing has been paid less attention. In this paper, we develop a family of test statistics which compete with other tests of general use, like the likelihood ratio test.

We consider the general mixed linear model

$$y = X\beta + Z_1 u_1 + \cdots + Z_s u_s + e, \tag{1}$$

where $y = (y_1, \ldots, y_n)^{\mathrm{t}}$ is the vector of observations of the response variable, $\beta = (\beta_1, \ldots, \beta_p)^{\mathrm{t}}$ is the vector of (unknown) fixed effects and $u_i = (u_{i1}, \ldots, u_{iq_i})^{\mathrm{t}}$ is the vector of effects of the q_i levels of the ith random factor,

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* Corresponding author.
E-mail addresses: isabel.molina@uc3m.es (I. Molina), d.morales@umh.es (D. Morales).

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$i = 1, \ldots, s$. Thus, in the sequel u_i itself will be referred to as the ith random factor. Finally, $e = (e_1, \ldots, e_n)^t$ is the vector of random errors, and $X, Z_1, \ldots, Z_s$ are known design matrices, of orders $n \times p, n \times q_1, \ldots, n \times q_s$, respectively.

The following assumptions listed by Miller (1977) ensure the estimability of all model parameters:

(F1) $u_1, \ldots, u_s, e$ are independent and normally distributed, more precisely $e \sim \mathcal{N}_n(0, \sigma_0^2 I_n)$ and $u_i \sim \mathcal{N}_{q_i}(0, \sigma_i^2 I_{q_i})$, $i = 1, \ldots, s$, where $\sigma_i^2, i = 0, 1, \ldots, s$, are unknown parameters typically called variance components.

(F2) $\text{rank}(X) = p$. (It can always be satisfied by suitable reparametrization of the model.)

The number of observations must be at least the number of parameters, that is

(F3) $n \geqslant p + s + 1$.

Another necessary assumption is that fixed effects cannot be confounded with the effects of any random factor, i.e., if $[A : B]$ denotes the matrix built by placing B on the right of A,

(F4) $\text{rank}[X : Z_i] > p, i = 1, \ldots, s$.

Let us define $G_i = Z_i Z_i^t, i = 1, \ldots, s$, and $G_0 = I_n$. (F5) assures that the effects of any random factor are not confounded with the effects of any other random factor.

(F5) $G_0, G_1, \ldots, G_s$ are linearly independent, that is,

$$\sum_{i=0}^{s} \alpha_i G_i = 0 \quad \text{implies } \alpha_i = 0, \quad i = 0, 1, \ldots, s.$$

Finally, (F6) determines the shape of $Z_i, i = 1, \ldots, s$.

(F6) Z_i contains only zeros and ones, in such a way that there exists exactly one 1 in each row, and at least one 1 in each column, $i = 1, \ldots, s$.

The previous assumption implies that $Z_i^t Z_i = D_i$, where D_i is a diagonal nonsingular matrix of order $q_i \times q_i$, that $\text{rank}(Z_i) = q_i$, and that $q_i \leqslant n, i = 1, \ldots, s$.

From previous assumptions, we deduce that

$$y \sim \mathcal{N}_n(X\beta, V), \quad \text{where } V = V(\sigma^2) = \sum_{i=0}^{s} \sigma_i^2 G_i \text{ and } \sigma^2 = (\sigma_0^2, \sigma_1^2, \ldots, \sigma_s^2)^t,$$

and it is required that

(F7) V is regular.

Let $\theta^t = (\beta^t, (\sigma^2)^t)$ denote the vector of unknown parameters, so that the parametric space is

$$\Theta = \{\theta^t = (\beta^t, (\sigma^2)^t); \beta \in \mathbb{R}^p; \sigma_0^2 > 0; \sigma_i^2 \geqslant 0, i = 1, \ldots, s\}. \tag{2}$$

For an observed y, the likelihood of θ is denoted in the same way as the joint probability density function (p.d.f.) of y given θ, i.e.

$$f_\theta(y) = (2\pi)^{-n/2} |V|^{-1/2} \exp\{-\tfrac{1}{2}(y - X\beta)^t V^{-1} (y - X\beta)\}, \tag{3}$$

and its loglikelihood by $l(\theta) = \ln f_\theta(y)$.