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Minimum aberration blocking schemes for two- and three-level fractional factorial designs

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Abstract

The concept of minimum aberration has been extended to choose blocked fractional factorial designs (FFDs). The minimum aberration criterion ranks blocked FFDs according to their treatment and block wordlength patterns, which are often obtained by counting words in the treatment defining contrast subgroups and alias sets. When the number of factors is large, there are a huge number of words to be counted, causing some difficulties in computation. Based on coding theory, the concept of minimum moment aberration, proposed by Xu [Statist. Sinica, 13 (2003) 691–708] for unblocked FFDs, is extended to blocked FFDs. A method is then proposed for constructing minimum aberration blocked FFDs without using defining contrast subgroups and alias sets. Minimum aberration blocked FFDs for all 32 runs, 64 runs up to 32 factors, and all 81 runs are given with respect to three combined wordlength patterns.

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Keywords: Blocking; Fractional factorial designs; Linear code; Minimum aberration; Minimum moment aberration; Wordlength pattern

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1. Introduction

Fractional factorial designs (FFDs) are widely used in designing experiments. Blocking is an effective method for reducing systematic variations and therefore increasing precision of effect estimation. Experimenters would often face the problem of choosing optimally blocked FFDs.

FFDs are typically chosen according to the *maximum resolution* criterion (Box and Hunter, 1961) and its refinement, the *minimum aberration* (MA) criterion (Fries and Hunter, 1980). The study of blocking in FFDs is complicated by the presence of two defining contrast subgroups, one for defining the fraction and another for defining the blocking scheme, and therefore, resulting in two types of wordlength patterns, one for treatment and another for block. Bisgaard (1994) proposed resolution for choosing blocked FFDs. However, resolution alone is not significant enough to rank order blocked FFDs. The MA criterion can be applied to the treatment and block wordlength patterns separately. However, MA designs with respect to one wordlength pattern may not have MA with respect to the other wordlength pattern. One approach, as done by Sun et al. (1997) and Mukerjee and Wu (1999), is to consider the concept of admissible blocking schemes, but it is often to have too many admissible designs. Another approach is to combine the treatment and block wordlength patterns into one single wordlength pattern so that the criterion of MA can be applied to it in the usual way; see Sitter et al. (1997), Chen and Cheng (1999), Zhang and Park (2000), and Cheng and Wu (2002).

Sun et al. (1997) provided collections of admissible blocked FFDs with 8, 16, 32, 64, and 128 runs up to 9 factors. Sitter et al. (1997) provided collections of MA blocked FFDs with all 8 and 16 runs, 32 runs up to 15 factors, 64 runs up to 9 factors, and 128 runs up to 9 factors. Chen and Cheng (1999) developed a theory to characterize MA blocked FFDs in terms of their blocked residual designs and gave collections of MA blocked FFDs with all 8 and 16 runs, and 32 runs up to 20 factors. Cheng and Wu (2002) compared MA blocked FFDs with respect to different combined wordlength patterns for 8, 16, 32, 64, and 128 runs up to 9 factors; they also provided collections of MA and admissible blocked FFDs with all 27 runs, and 81 runs up to 10 factors.

The MA criterion ranks blocked FFDs according to the treatment and block wordlength patterns, which are often obtained by counting words in the treatment defining contrast subgroups and alias sets. When the number of factors is large, there are a huge number of words to be counted, causing some difficulties in computation. For example, when an FFD with 32 runs and 20 factors is arranged in 8 blocks, there are $2^{15} - 1 = 32,767$ words in the treatment defining contrast subgroup and 7 block effects, each block effect being confounded with $2^{15} = 32,768$ treatment effects. When an FFD with 81 runs and 20 factors is arranged in 27 blocks, there are $(3^{16} - 1)/(3 - 1) = 21,523,360$ words in the treatment defining contrast subgroup and 13 block effects, each block effect being confounded with $3^{16} = 43,046,721$ treatment effects. It is very time consuming to count all these words or effects. This explains, partially at least, why MA blocked FFDs are available only up to 9 or 10 factors in most cases in the literature.

The purpose of this paper is to construct more MA blocked FFDs with a large number of factors. This is challenging due to aforementioned computational difficulties. Based on coding theory, we propose new methods to compare and rank blocked FFDs without using