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Bayesian sensitivity analysis and model comparison for skew elliptical models ☆

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Abstract

In this work we approach the problem of model comparison between skew families. For the univariate skew model, we measure the sensitivity of the skewness parameter using the L_1 distance between symmetric and asymmetric models and we obtain explicit expressions for some of these models. The main result is that the L_1 distance between a representable elliptical distribution and a representable skew elliptical distribution remains invariant and it equals to the L_1 distance between the normal and skew-normal densities. We also use the Bayes factor to test asymmetry and present some simulation results for the skew-normal and skew-*t* distributions obtaining expected results for adequate prior distribution. An application in stock markets is also considered.

MSC: 62F15; 62F03

Keywords: Skew distribution; Skew-normal distribution; Representable skew elliptical distribution; Bayes factor; L_1 distance

1. Introduction

In a substantial number of applications symmetric (elliptical) models, in particular the normal model, have been found to be restrictive and more realistic models are needed.

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The main focus in the present work is when the random quantities exhibits skewness. Recently, new alternative models have been developed with the goal of preserving the good properties of the elliptical models and also to be more flexible to model more realistically a data set. These more general models allow us to control the skewness and the kurtosis of the distribution and it includes the normal case as a special one. However, these more flexible models increase the mathematical complexity. Computational techniques can solve partially the problem, even though, some mathematical calculation needs to be done in order to obtain accurate results.

The idea proposed by Azzalini (1985) in the context of the normal distribution, introduce skewness in a symmetric distribution in the following way. If f and g are symmetric probability density function (pdf) around zero and G is a continuous cumulative distribution function (cdf) associated with g, then

$$\frac{2}{\sigma} f\left(\frac{x-\mu}{\sigma}\right) G\left(\lambda \frac{x-\mu}{\sigma}\right) \tag{1}$$

is a skew pdf for any $\lambda \in \mathbb{R}$, where $\mu \in \mathbb{R}$ is a location parameter, $\sigma > 0$ is a scale parameter and λ is a skewness parameter. When $\lambda = 0$ we obtain the symmetric location–scale pdf, $(1/\sigma) f((x-\mu)/\sigma)$. Different choices of the f and G functions give us important special cases. For example, if $f = \phi$ and $G = \Phi$, the pdf and cdf of normal distribution respectively, we have the skew-normal distribution which is denoted by $\mathrm{SN}(\lambda, \mu, \sigma)$.

The elliptical model given by Kelker (1970) is another well-known generalization of the normal model. This model has been studied, for example, by Cambanis et al. (1981), Fang et al. (1990) and Arellano-Valle (1994). The elliptical model includes a vast variety of important distributions (the Student-t distribution, double exponential, Pearson type II) and also has good properties, for example they are closed under marginalization and conditioning. The symmetry of the normal model is preserved, but different kurtosis coefficient are allowed.

Extension of the normal model connecting the two ideas, skewness and heavy tails, have been studied by Branco and Dey (2001) and Genton and Loperfido (2001). An interesting special case is the skew-t distribution, which could be represented by a mixture of skew-normal models.

In this paper we approach the problem of model comparison within skew elliptical families. In Section 2, we measure the sensitivity of the skewness parameter using the L_1 distance between the symmetric and asymmetric models. Computation of the Bayes factor to examine asymmetry is presented in Section 3. Also, in Section 4 we present simulation results for the skew-normal and skew-t distributions. An application in Chilean's stock markets is also considered.

2. Sensitivity analysis for the skewness parameter

In this section, we analyze the sensitivity of the skewness parameter on the model given by (1), by measuring this sensitivity using the L_1 distance between the following