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Testing the information matrix equality with robust estimators

Christophe Croux*, Geert Dhaene, Dirk Hoorelbeke

Faculty of Economics and Applied Economics, Katholieke Universiteit Leuven, Naamsestraat 69, Leuven B-3000, Belgium

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Abstract

The information matrix (IM) equality can be used to test for misspecification of a parametric model. We study the behavior of the IM test when the maximum-likelihood (ML) estimators used in the construction of this test are replaced with robust estimators. The latter do not suffer from the masking effect in the presence of outliers and can improve the power of the IM test. At the normal location-scale model, the IM test using the ML estimators is known as the Jarque–Bera test, and uses skewness and kurtosis to detect deviations from normality. When robust estimators are employed to test the IM equality, a robust version of the Jarque–Bera test emerges. We investigate in detail the local asymptotic power of the IM test, for various estimators and under a variety of local alternatives. For the normal regression model, it is shown by simulations under fixed alternatives that in many cases the use of robust estimators substantially increases the power of the IM test.

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1. Introduction

White (1982) introduced the information matrix (IM) test as an omnibus test for misspecification of a parametric model. The test exploits the well-known property that, if the model

* Corresponding author. Tel.: +32 16 326958; fax: +32 16 326732.
E-mail address: christophe.croux@econ.kuleuven.ac.be (C. Croux).

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is correctly specified, the sum of the Hessian of the log-likelihood and the outer product of the score has zero expectation. So if at parameter estimates the sample average of the sum of the Hessian and the outer product of the score differs significantly from zero, this is evidence against the model. The IM test is typically implemented using maximum-likelihood (ML) estimates of the parameters. In this paper, we explore the potential gain of replacing the ML estimator with a robust estimator. It will be shown that this often leads to a gain in power.

When the IM test is used in conjunction with the ML estimator, the test may suffer from the masking effect. Outlying observations will not be recognized as such (or less so, compared to robust estimators), and hence the test will have relatively low power against distributions with fat tails or when outliers are present. One may thus conjecture that unmasking outliers, by using robust estimators, will increase the power of the IM test. In most of the cases that we consider, using robust estimators effectively increases the power of the IM test.

We show that when robust estimators are used, the IM test statistic (being a quadratic form) still has a limiting χ^2 distribution under the null hypothesis. A closed-form expression for the asymptotic covariance matrix V, to be used in the IM statistic, is derived in a general setting.

We analyze the normal location-scale model in detail. For the ML estimator the IM test is the Jarque–Bera (1980) test of normality. The Jarque–Bera test combines the skewness and kurtosis coefficient to detect deviations from normality. When using the ML estimator the rank of V equals 2, while for any other M-estimator of scale, the rank of V equals 3, indicating that in this case the IM test is sensitive to 'more' specification error, namely not only the third and fourth moment are tested now, but also the first two. Explicit expressions for V are derived for two robust M-estimators of scale, namely the M-estimator based on Tukey's biweight (TB) function and the median absolute deviation (MAD).

In the linear regression model with normal error terms the IM test is a combined test for heteroscedasticity, skewness and non-normal kurtosis (Hall, 1987). We use an S-estimator (Rousseeuw and Yohai, 1984) or an MM-estimator (Yohai, 1987) as robust estimators of regression, and an M-estimator based on TB function as a robust estimator of residual scale. It is well-known that the ML estimator tends to mask outlying observations, and this danger is more severe in the regression model than in the location-scale model. It is therefore to be expected that the use of robust estimators holds more promise in the regression case. Simulation results indeed indicate that using robust estimators increases the power of the IM test in the case of a thick-tailed alternative like the Cauchy distribution. In the presence of outliers, there is even a dramatic gain in power from using robust estimators.

We present analytic local power results for the IM test in the normal location-scale model under four different sequences of local alternatives: (i) a contaminated normal; (ii) Student's t; (iii) a skewed normal; and (iv) a tilted normal. The asymptotic distribution of the IM statistic is, as usual, non-central χ^2. Furthermore, when outlying observations are present, IM tests that use robust estimators (such as the 25% breakdown point TB estimator, which strikes a balance between high robustness and high efficiency) are shown to be much more powerful than the IM test that uses the ML estimator. On the other hand, when the local alternative is Student's t (with degrees of freedom approaching infinity), skewed normal, or tilted normal, the non-centrality parameter is identical for all M-estimators. We derive closed-form expressions of the non-centrality parameter under any local alternative