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A nonparametric procedure based on early failures for selecting the best population using a test for equality

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Abstract

In this paper, we first propose a nonparametric test based on early failures for the equality of two lifetime distributions against two alternatives regarding the best population. This procedure makes use of the precedence statistic which can determine the difference between populations based on early ($100q\%$) failures. Hence, this procedure can be useful in life-testing experiments in biological as well as industrial settings. First, we propose the test procedure and derive the null distribution of the test statistic in the two-sample case with equal or unequal sample sizes. We also present the exact probability of correct selection under the Lehmann alternative. Then, we generalize the test procedure to the k-sample case. Critical values for some sample sizes are presented. Next, we examine the performance of this test procedure under a location-shift alternative through Monte Carlo simulations. An example is presented to illustrate our test procedure with selecting the best population as an objective. Finally, we make a few concluding remarks and some suggestions for the choice of q.

Keywords: Two-sample problem; k-sample problem; Precedence statistics; Life-testing; Lehmann alternative; Monte Carlo simulations; Probability of correct selection

1. Introduction

The precedence test is a distribution-free two-sample life test based on the order of early failures, and was first proposed by Nelson (1963) in order to test for the equality of two life-time distributions. After that, many authors have studied the power properties of the precedence test

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and also proposed some alternative tests; see, for example, Eilbott and Nadler (1965), Shorack (1967), Nelson (1986, 1993), Lin and Sukhatme (1992), Chakraborti and van der Laan (1996, 1997), Balakrishnan and Frattina (2000), Balakrishnan and Ng (2001), and van der Laan and Chakraborti (2001). Wilcoxon-type rank-sum precedence tests have been constructed by Ng and Balakrishnan (2002, 2004), who have also examined the power properties in comparison to the precedence test. Chakraborti and van der Laan (1997) considered the precedence test to compare several treatments with a control based on censored data. Precedence tests will be useful (i) when life-tests involve expensive units since the units that had not failed could be used for other purposes, and (ii) to make quick and reliable decisions early on in the life-testing experiment having observed only a few failures. All the precedence tests considered by earlier authors were carried out against a one-sided alternative or a two-sided (global) alternative. In extending the test to k (>2) populations, a global alternative and a restricted (simple ordered) alternative previously have been considered.

When we have two or more competing populations, a problem of interest often is to make a decision as to which is the best if we reject the null (homogeneity) hypothesis. To this end we formulate the problem as a multiple-decision problem. We still control the probability of rejecting a true null hypothesis. However, we use the probability of selecting the best population as a performance characteristic. It is important to note that ours is not the usual ranking and selection formulation which does not have a null hypothesis and typically controls the probability of selecting the best population. For a general introduction to ranking and selection problems, see Bechhofer et al. (1995), Gibbons et al. (1999) and Gupta and Panchapakesan (2002).

This paper is organized as follows. In Section 2, we introduce a nonparametric test procedure to test the equality of two distributions against two alternative hypotheses that terminates the experiment with no more than $100q\%$ of the two samples have failed. The test statistic is proposed and its null distribution is derived. Both equal and unequal sample sizes cases are considered and critical values close to 5% level of significance are presented. Exact probability of correct selection of these tests under the Lehmann alternative is also discussed. In Section 3, we extend this test procedure to the k-sample case. We discuss the test statistic and derive its null distribution. We examine the properties of the test procedure under a location-shift between the populations through Monte Carlo simulations for $k=2$ and 3. Next, we discuss the simulation results and the choice of the value of q in Section 4. Finally, we present an example in Section 5 to illustrate the procedures discussed in the preceding sections.

2. Two-sample problem

Suppose there are independent samples coming from two different populations. Let $X_1, X_2, \ldots, X_{n_1}$ denote the sample from population π_1 with distribution F_X, and $Y_1, Y_2, \ldots, Y_{n_2}$ denote the sample from population π_2 with distribution F_Y. The number of units under the life-testing experiment is thus $N=n_1+n_2$. These two groups of units are placed on a life-test simultaneously. We are concerned with the equality of the two populations and the problem of selecting the best population among π_1 and π_2 if they are not equal. This problem is, of course, related to testing the hypothesis

$$\mathrm{H}_0 : F_X = F_Y \quad \text{vs.} \quad \begin{cases} \mathrm{H}_{A1} : F_X < F_Y, \\ \mathrm{H}_{A2} : F_X > F_Y. \end{cases} \tag{2.1}$$