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LAN theorem for non-Gaussian locally stationary processes and its applications

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Abstract

For a class of locally stationary processes introduced by Dahlhaus, we derive the LAN theorem under non-Gaussianity and apply the results to asymptotically optimal estimation and testing problems. For a class $\mathscr{F}$ of statistics which includes important statistics, we derive the asymptotic distributions of statistics in $\mathscr{F}$ under contiguous alternatives of unknown parameter. Because the asymptotics depend on the non-Gaussianity of the process, we discuss the non-Gaussian robustness. An interesting feature of effect of non-Gaussianity is elucidated in terms of LAN. Furthermore, the LAN theorem is applied to adaptive estimation when the innovation density is unknown.

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Keywords: Locally stationary process; Local asymptotic normality; Asymptotically efficient estimator; Optimal test; Non-Gaussian robustness; Adaptive estimation

1. Introduction

Time series analysis under stationary assumption is well established. The assumption of stationarity guarantees that the increase of sample size provides more and more information of the same kind which is basic for an asymptotic theory to make sense. However it is not sufficient for stationary time series models to describe the real world. Many empirical studies

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show that real time series data are generally non-stationary. Therefore the assumption of non-stationarity is natural in time series analysis. When we deal with non-stationary processes, one of the difficult problems to solve is how to set up an adequate asymptotic theory. Recently, to overcome this problem, Dahlhaus (1996a, b, 1997) proposed an important class of locally stationary processes and elucidated some fundamental results of the statistical inference.

In statistical asymptotic theory local asymptotic normality (LAN) (see e.g. LeCam, 1986; Strasser, 1985) is one of the most fundamental concept and describes the optimal solution of virtually all asymptotic inference and testing problems. The LAN approach has been introduced in time series analysis. Swensen (1985) established LAN for autoregressive models of finite order with a regression trend and applied it in the derivation of the local power of the Durbin–Watson test. Kreiss (1987, 1990) also proved the LAN property for ARMA and AR(∞) models, and constructed locally asymptotically minimax LAM adaptive estimators, as well as locally asymptotically maximin tests. For time series regression models with long memory disturbance, Hallin et al. (1999) showed LAN theorem and discussed an adaptive estimation. Taniguchi and Kakizawa (2000) gave an extensive review for LAN approach in time series analysis.

For non-stationary case, Dahlhaus (1996b) developed asymptotic theory for Gaussian locally stationary processes and derived the LAN property. In this paper we drop the Gaussian assumption, i.e., we prove the LAN theorem for non-Gaussian locally stationary processes, and apply the results to the asymptotic estimation and testing theory. Our LAN theorem elucidates the non-Gaussian asymptotics and the proof and method contain a lot of different parts from Gaussian case. We discuss a non-Gaussian robustness for a class of statistics which have quadratic forms. The class includes the main order term of QMLE, tests and discriminant statistics etc. Hence, in this sense it contains a lot of important statistics. We are often interested in the local asymptotics under $\theta_T = \theta + \frac{h}{\sqrt{T}}$. In view of LeCam's third lemma, together with the LAN results we give the limit distributions of the quadratic forms under θ_T, which depend on non-Gaussian quantities. If they are independent of the non-Gaussian quantities, we say that they are non-Gaussian robust. Then we give a set of sufficient conditions for the non-Gaussian robustness, which illuminate new scope of non-Gaussian approach. Also some numerical studies are given, and show unexpected features. Finally, we apply the LAN theorem to construct adaptive estimators. When the innovation density is unknown, the asymptotically efficient estimator is given, as well as when the innovation density is known.

The paper is organized as follows. Section 2 explains the locally stationary processes with unknown parameter, and provides the likelihood ratio between contiguous parameter hypotheses. In Section 3 we prove the LAN theorem. Based on this, asymptotically optimal estimator and test statistic are given. Section 4 addresses a non-Gaussian robustness for a class $\mathscr{F}$ of statistics, which includes the main order term of QMLE, tests and discriminant statistics. We derive the asymptotic distributions for statistics in $\mathscr{F}$ under contiguous alternatives. Because they depend on non-Gaussianity of the process concerned, we illuminate when they are independent of the non-Gaussianity. From this some interesting features are observed. Section 5 discusses adaptive estimation when the innovation density is unknown. Using an estimated innovation density, we construct an asymptotically efficient (adaptive) estimator. Proofs of theorems and the related lemmas are placed in Section 6.