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Normalizing unbiased estimating functions

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Abstract

We consider a method for setting second-order accurate confidence intervals for a scalar parameter by applying normalizing transformations to unbiased estimating functions. Normalizing a nonlinear estimating function is usually easier than normalizing the estimator defined as the solution to the corresponding estimating equation. This estimator usually has to be obtained by some iterative algorithm. Numerical examples include a canonical Poisson regression and the estimation of the correlation coefficient. Numerical comparisons are made with the asymptotically equivalent method called estimating function bootstrap proposed recently by Hu and Kalbfleisch (Canad. J. Statist. 28 (2000) 449).

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1. Introduction

Let $Y_1, \ldots, Y_n$ be independent discrete or continuous random variables having possibly different distributions. Assume that these distributions depend on a common scalar

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parameter θ, inference about which is to be based on the following unbiased additive estimating function:

$$g(\theta) = \sum_{i=1}^{n} g_i(Y_i, \theta), \tag{1.1}$$

where $E_\theta\{g_i(Y_i, \theta)\} = 0$ for any $\theta \in \Theta$ and $i = 1, \ldots, n$. The estimating function (1.1) may arise in the maximum likelihood estimation, the quasi-likelihood inference, the weighted least-squares estimation, etc. The root $\hat{\theta}$ to the estimating equation $g(\hat{\theta}) = 0$, assumed unique in this paper (see e.g. Small and Wang (2003) for discussions on estimating equations with multiple roots), defines an estimator of θ. Under mild conditions (e.g., Crowder, 1986), $\hat{\theta}$ is $\sqrt{n}$-consistent for the true value of θ. Only in exceptional cases, there will exist an analytical formula for $\hat{\theta}$. Usually, $g(\hat{\theta}) = 0$ will have to be solved using some iterative methods, such as the iteratively reweighted least squares method (Green, 1984) for generalized linear regressions.

In this paper, we consider a simple analytical method for obtaining an approximate confidence distribution for θ based on $g(\theta)$ of (1.1). The associated confidence intervals $\hat{I} = (-\infty,\ w_\alpha)$ are shown to be *second-order accurate*, meaning that

$$\Pr\{\theta \leqslant w_\alpha\} = 1 - \alpha + \mathrm{o}(n^{-1/2}),$$

where $1 - \alpha$ is the nominal coverage level and n the sample size. The confidence limit w_α is defined by the solution to

$$s_\lambda(w_\alpha) = z_\alpha, \quad (\Phi(z_\alpha) = \alpha),$$

where $\Phi(\cdot)$ is the CDF of the unit normal variate $\mathrm{N}(0, 1)$, and $s_\lambda(\theta)$ the normalized estimating function defined by (2.8).

There is a large literature on setting accurate confidence intervals. Most methods however treat as the primal an estimator, which is assumed to be available in *closed form*. The nonparametric plug-in estimators are such examples. Given such an estimator, and other conditions on moments, second-order accurate intervals can be obtained by using Cornish–Fisher expansions (Barndorff–Nielsen and Cox, 1989), or asymptotically equivalent methods, such as inverse Edgeworth expansions (e.g., Hall, 1983) or theories of normalizing transformations (e.g., Fisher, 1921; Konishi, 1981, 1991). Alternatively, computationally more intensive methods, such as the $\mathrm{BC_a}$ bootstrap (Efron, 1987), also lead to second-order accurate intervals. When an estimator is available in closed form, the bootstrap is much to recommend because good approximation to the bootstrap distribution of the estimator can be computed with relative ease.

When an estimator is defined in an iterative manner, as is the case with (1.1), the above methods are either difficult to apply or computationally too costly. A different strategy is to treat as the basic element the given estimating function $g(\theta)$ rather than the estimator $\hat{\theta}$ defined by $g(\hat{\theta}) = 0$. The idea is to associate a confidence distribution with the parameter of interest using the properties of an unbiased estimating function, an idea which may be traced back to the work of Wilks (1938), who proved, among other things, the asymptotic optimality of the score function in terms of the expected length of confidence intervals. The