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Review

Asymptotics of the signed-rank estimator under dependent observations

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ABSTRACT

In this paper, we consider a signed-rank estimator of nonlinear regression coefficients under stochastic errors. These errors include a wide array of applications in economic literature such as serial correlation, heteroscedasticity, autoregression, etc. General conditions for strong consistency and $\sqrt{T}-$asymptotic normality of the resulting estimator are provided.

Contents

1. Introduction

Nonlinear regression models play a central role in the empirical analysis of economic phenomena. They have been shown to be very useful in empirical macroeconomics and microeconomics. Also they are used in the explanation or description of data encountered by economists in multiple forms such as time-series, cross-sectional, panel, and experimental data. A common inferential problem for these regression models involves estimating the true regression parameters. Several approaches to solve this problem have been investigated in the literature including the least squares (LS) and the least absolute deviation (LAD) procedures.

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This paper is concerned with the study of a general signed-rank estimator of the regression coefficients that is obtained as a minimizer of an objective function $D_T(\theta)$ defined by Eq. (2.2) in Section 2. The motivation for using the signed-rank approach comes from the fact that it provides robust estimators compared to the most common LS approach when dealing with situations where there are outlying observations in the response space and/or where the error distribution is heavy-tailed. A particular case of the signed-rank approach (taking $\varphi = 1$ in Eq. (2.2)), the LAD estimator, has been studied by Oberhofer (1982) and Haupt and Oberhofer (2009). Although robust in many situations, the LAD has been shown to be less efficient than the estimator commonly referred to as the signed-rank estimator found by taking $\varphi(t) = t$ in Eq. (2.2) (Hettmansperger and McKean, 1998). The general form of the signed-rank estimator defined as a minimizer of Eq. (2.2) has been extensively studied in the literature by authors such as Hettmansperger and McKean (1998), Bindele and Abebe (2012), and Abebe et al. (2012) for models with i.i.d. errors and by Mukherjee (1999) for linear autoregressive models. Applications of this approach to factorial designs under dependent observations are given in Brunner and Denker (1994). As the signed-rank procedure is based on rank scores, an important approach using such rank scores in estimating the true nonlinear regression parameter is developed in Jurečková (2008) for i.i.d. errors and extended to time-series models by Mukherjee (1999). Mukherjee (1999) provided a comprehensive review of different techniques of estimation in the literature including M-estimation, R-estimation and L-estimation.

Such theoretical asymptotic properties (consistency and asymptotic normality) least squares (LS) estimators for the nonlinear models with fixed regressors and i.i.d. errors were established by Jennrich (1969), Malinvaud (1970), Wu (1981), Gallant and Gallant (1973), and Burguete et al. (1982), among others. Extension of these results to time-series data allowing for stationary, ergodic errors, was provided by Hannan (1971) whose results were generalized by Robinson (1972) to systems of equations. General conditions for the asymptotic properties of the LS estimator for most situations encountered in economics are discussed in White and Domowitz (1984). An LS approach allowing the errors to be martingale differences is provided in Skouras (2000). Also, that of allowing martingale differences errors and requiring the dependent and explanatory variables to be jointly strictly stationary and ergodic were studied by Sims (1976). An approach allowing explanatory variables to be uniform mixing processes with i.i.d. errors is given in Bierens (1982). Lai and Wei (1982) established the asymptotic properties of the LS estimator in stochastic regression models. Other works that studied the consistency of the LS estimator include Anderson and Taylor (1979), Christopeit and Helmes (1980), and Andrews (1986).

In the framework of general nonlinear regression with dependent error structure, to our best knowledge, no asymptotic results have been provided in for the general signed-rank estimator. The purpose of this paper is to provide conditions needed for the strong consistency and the $\sqrt{T}-$asymptotic normality of the resulting estimator toward statistical inference for dependent discrete time stochastic errors, specifically when the errors are either ϕ-mixing or α-mixing sequence of random processes.

To this end, the rest of the paper is organized as follows: in Section 2, we define the model and provide the estimation procedure of the regression coefficients. Sections 3 and 4 are concerned with the asymptotic properties of the signed rank estimator. A conclusion summarizing the findings of the paper is given in Section 5. Finally, Appendix A contains proofs of some results in the paper.

2. Model definition and estimation

Consider the general stochastic regression model

$$y_t = f_t(\mathbf{x}_t, \theta_0) + \varepsilon_t, \quad 1 \le t \le T, \tag{2.1}$$

with the following settings: If $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ represents a filtered probability space, $f_t : \mathbb{R}^d \times \mathbf{\Theta} \to \mathbb{R}$ such that $f_t(\mathbf{x}_t, \theta) = f_t(\theta)$ is a measurable known function on $\mathbb{R}^d$ for each $\theta \in \mathbf{\Theta}$ and twice continuously differentiable on $\mathbf{\Theta} \subset \mathbb{R}^p$ (compact), uniformly on t with probability 1. $\theta_0 \in Int(\mathbf{\Theta}) \subset \mathbb{R}^p$ is the true vector of parameters, y_t and $\mathbf{x}_t$ are observable but ε_t is not. $\{\varepsilon_t\}_{t \ge 1}$ is a ϕ-mixing or α-mixing sequence of identically distributed random processes. Although the α-mixing condition is more general, it is sometimes possible to put weaker restrictions on the speed at which $\phi(T)$ approaches zero than for $\alpha(T)$. As discussed in White and Domowitz (1984), most of the processes which are time-dependent as presented in Anderson (1958) satisfy the ϕ-mixing condition, as do certain finite order ARMA processes. Finite order Gaussian ARMA processes are shown to be α-mixing (Ibragimov and Linnik, 1971). General class of models with many economic applications is that of stationary Markov chains (Lee et al., 1977) and Rosenblatt (1956) provides general conditions under which stationary Markov processes are α-mixing. The class of models described by (2.1) is very wide, and includes many commonly used linear and nonlinear regression models. For example, the $f_t(\theta)$'s can be linear or nonlinear functions of the past observations $(y_1, \ldots, y_{T-1})$, and any other covariates $\mathbf{x}_t = (x_1, \ldots, x_t)$ such that x_t is $\mathcal{F}_{t-1}$-measurable. Important classes of such models include nonlinear times series models (with or without exogenous inputs), stochastic control models, stochastic approximate schemes and sequential designs. This includes the NARX model discussed as example in Lai and Wei (1982). Examples of the function $f_t(\theta)$ can be found in Skouras (2000), White and Domowitz (1984), and Lai and Wei (1982). Other examples of such regression models with heteroscedastic MA errors or heteroscedastic random coefficient AR error processes are given in Cragg (1982) and Weiss (1986).

Also, as pointed out in White and Domowitz (1984), two interpretations on the specification of (2.1) are possible. $f_t(\theta)$ may be regarded as a physical response function, making y_t to be the observed response to $\mathbf{x}_t$, incorporating additive errors's measurement or random shocks, ε_t. Also, model (2.1) can be regarded as a conditional expectation with respect to the