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Accelerated life test planning with independent lognormal competing risks

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ABSTRACT

In accelerated life testing (ALT), products are exposed to stress levels higher than those at normal use in order to obtain information in a timely manner. Past work on planning ALT is predominantly on a single cause of failure. This article presents methods for planning ALT in the presence of k competing risks. Expressions for computing the Fisher information matrix are presented when risks are independently distributed lognormal. Optimal test plans are obtained under criteria that are based on determinants and maximum likelihood estimation. The proposed method is demonstrated on ALT of motor insulation.

1. Introduction

Accelerated life testing (ALT) exposes products to higher than usual levels of stress, e.g. temperature, voltage, and use rate, in order to quickly obtain lifetime information. Because of constraints on time and resources, ALT provides an economic way of obtaining lifetime data. ALT data are often collected to estimate, e.g. by maximum likelihood (ML), a lifetime quantile at the level that the product is normally used, the failure rate at a given level, or several model parameters. A test plan that performs well, say, for estimating a life quantile may not perform well in estimating model parameters. Thus, careful design of ALT is imperative.

Most of the past work on ALT planning assumed that there is a single cause of failure. For example, Nelson and Kielpinski (1976), Nelson and Meeker (1978), and Nelson (1990) provided expressions for the Fisher information matrix when failure times were distributed lognormal or Weibull. For these situations, Meeker and Hahn (1985) recommended the 4:2:1 design (i.e. experimental unit allocations of 4/7, 2/7, 1/7 to three equally spaced increasing levels). Nelson (2005a, 2005b) described the process of planning ALT and the different methods for analyzing ALT data. He also enumerated relevant journal articles and books on the topic.

A test unit or product may fail due to one of several causes, called competing risks or failure modes. Chapter 7 of Nelson (1990), Kim and Bai (2002), Craiu and Lee (2005) described situations in engineering when competing risks occurred. For example, motor insulation may fail because of Turn, Ground and Phase failures. Semiconductor devices may fail because of

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failures of the lead or junction. A race or ball failure causes ball bearing assemblies to malfunction. There are a number of articles on the analysis of competing-risk data some of which are reviewed in Bunea and Mazzuchi (2003) and Pascual (2007, 2008b).

While a significant amount of work has been done on designing ALT for a single failure mode, there is a scant amount of discussions of ALT designs for 2 or more failure modes. For example, Bai and Chun (1991) described competing risks with independent exponential distributions and derived optimal step-stress ALT designs. Pascual (2007, 2008a) considered ALT designs when failure modes were independently distributed Weibull with known and common shape parameters. Pascual (2008b) studied the more general case when failures modes had different and unknown Weibull shape parameters. Expressions for the Fisher information matrices were derived in the latter three references.

The Weibull and lognormal distributions are two of the most popular distributions used in statistical analysis of reliability data. Nelson (1990, p. 63) remarked that when both these distributions are fit to the same data, predictions under the Weibull distribution tend to be more "pessimistic at the lower tails" than under lognormal. That is, Weibull estimates of lower quantiles tend to be smaller than those of lognormal's. Because practitioners are often interested in lower lifetime quantiles, the choice of the distribution is critical. Also, the expressions for the lognormal Fisher information are expected to be different from those for Weibull ALT planning in Pascual (2008b). Thus, lognormal test plans are expected to be different from Pascual's (2008b) plans.

This article explores the problem of designing ALT when failure modes may be described by independent lognormal distributions. Section 2 discusses model assumptions and provides corresponding expressions for the likelihood function for competing-risks data. Section 3 discusses the Arrhenius model that describes lifetime under temperature acceleration. Sections 4 and 5 provide expressions for the Fisher information matrix and ALT design criteria based upon it. The proposed methods are applied to designing ALT of Class-B insulation of motors in Section 7. Equivalence theorems for verifying optimality of designs are presented in Appendix A.

2. Model assumptions and likelihood function for independent lognormal risks

Suppose there are m unique experimental conditions, and let $\boldsymbol{s}_r$ represent the r th condition. Assume that test units can fail due to one of k defined risks and that the actual cause of failure can be identified by conducting an "autopsy" on the failed unit. In this section, $i=1,\ldots,k$ and $r=1,\ldots,m$. Let $T_{ij}^{(r)}$ be the random variable denoting the failure time of a test unit j due to risk i at condition $\boldsymbol{s}_r$. In the absence of censoring, unit j's lifetime is $T=\min\{T_{1j}^{(r)},\ldots,T_{kj}^{(r)}\}$. Assume that $T_{1j}^{(r)},\ldots,T_{kj}^{(r)}$ are independent. Thus, the life of a test unit is interpreted as that of a series system with k independent components.

Throughout this article log denotes natural logarithm. Assume that $T_{ij}^{(r)}$ is distributed lognormal, i.e. $\log[T_{ij}^{(r)}]$ is distributed normal, with location and scale parameters $\mu_i(s_r)$ and σ_i, respectively. Note that the location varies with test condition s_r according to a relationship to be defined later, while the scale does not. Throughout this article, ϕ, Φ and $\overline{\Phi}=1-\Phi$ denote the probability density function (pdf), cumulative distribution function (cdf), and survival function, respectively, of the standard normal distribution.

Let n_{ir} be the number of units that failed at $\boldsymbol{s}_r$ due to risk i. Let $t_{ij}^{(r)}$ be the failure time due to risk i at condition $\boldsymbol{s}_r$ for $j=1,\ldots,n_{ir}$. Let $n_{k+1,r}$ be the number of right censored lifetimes at $\boldsymbol{s}_r$, and let $t_{k+1,j}^{(r)}$ for $j=1,\ldots,n_{k+1,r}$ denote the corresponding censoring times. Note that a total of $n_{1r}+\cdots+n_{kr}+n_{k+1,r}$ units are tested at $\boldsymbol{s}_r$.

Following the approach in David and Moeschberger (1978), it can be shown that the loglikelihood contribution of all observations at condition $\boldsymbol{s}_r$ can be written as

$$\mathcal{L}_r(\boldsymbol{\theta})=-\sum_{i=1}^{k}n_{ir}\log(\sqrt{2\pi}\sigma_i)-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{n_{ir}}\left[\frac{\log(t_{ij}^{(r)})-\mu_i(\boldsymbol{s}_r)}{\sigma_i}\right]^2+\sum_{i,l=1;l\neq i}^{k}\sum_{j=1}^{n_{lr}}\log\left\{\overline{\Phi}\left[\frac{\log(t_{lj}^{(r)})-\mu_i(\boldsymbol{s}_r)}{\sigma_i}\right]\right\}$$
$$+\sum_{i=1}^{k}\sum_{j=1}^{n_{ir}}\log\left\{\overline{\Phi}\left[\frac{\log(t_{k+1,j}^{(r)})-\mu_i(\boldsymbol{s}_r)}{\sigma_i}\right]\right\}.$$

Thus, the total loglikelihood is

$$\mathcal{L}(\boldsymbol{\theta}|\boldsymbol{s}_1,\ldots,\boldsymbol{s}_m)=\sum_{r=1}^{m}\mathcal{L}_r(\boldsymbol{\theta}),$$

and the value $\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}$ that maximizes $\mathcal{L}$ is the maximum likelihood (ML) estimate of $\boldsymbol{\theta}$.

3. The Arrhenius model for temperature acceleration

Temperature is a variable commonly accelerated in ALT. The Arrhenius model is based on physical or chemical properties, and is often used to describe the failure process in this case. For more details, see Chapter 2 of Nelson (1990) and Chapter 18 of Meeker and Escobar (1998).