Journal of Statistical Planning and Inference 140 (2010) 454–460

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

Ordering conditional general coherent systems with exchangeable components

Zhengcheng Zhang

School of Mathematics, Physics and Software Engineering, Lanzhou Jiaotong University, Lanzhou, Gansu 730070, People's Republic of China

ARTICLE INFO

Article history:
Received 30 January 2009
Received in revised form
18 July 2009
Accepted 23 July 2009
Available online 11 August 2009

MSC:
60K10
60E15
62G30

Keywords:
Exchangeable
Hazard rate order
Inactivity time
Likelihood ratio order
Residual life
Reversed hazard rate order
Signature
Stochastic order

ABSTRACT

This paper investigates some ordering properties of the *residual lives* and the *inactivity times* of coherent systems with dependent exchangeable absolutely continuous components, based on the stochastically ordered *signatures* between systems, extending the results of Li and Zhang [2008. Some stochastic comparisons of conditional coherent systems. Applied Stochastic Models in Business and Industry 24, 541–549] for the case of independent and identically distributed components.

1. Introduction

Let T be the lifetime of a system with distribution function H, density function h and reliability function $\bar{H}$. Then, for any $t \geq 0$, the reliability function and density function of the residual life $(T-t|T>t)$ can be represented as $\bar{H}_t(x)=\bar{H}(t+x)/\bar{H}(t), h_t(x)=h(t+x)/\bar{H}(t)$, and the reliability function and density function of the inactivity time $(t-T|T\leq t)$ can be represented as $\bar{H}_{(t)}(x)=H(t-x)/H(t), h_{(t)}(x)=h(t-x)/H(t)$, for $x>0$.

For some coherent systems of order n, the system is still functioning with probability one when less than $r(<n)$ components fail at time $t>0$, and an operator may consider doing some maintenance, replacing the system, or comparing performances of competing systems in terms of residual life. As a dual notion to the residual life, inactivity time, measures the time elapsed since the system failure. It usually has a close connection with the so-called autopsy data, i.e. information obtained by examining the component states of a failed system.

In the literature, many authors paid their attentions to the residual lives and inactivity times of some coherent systems, especially k-out-of-n systems. The mean residual life of a parallel system under condition that none of the components has failed at time $t>0$ is first introduced in the work Bairamov et al. (2002) and developed in Asadi and Bairamov (2005, 2006). For the

E-mail addresses: zhangzc@mail.lzjtu.cn, zhzhcheng004@163.com.

0378-3758/$ - see front matter
doi:10.1016/j.jspi.2009.07.029

recent results on the residual lives and inactivity times of k-out-of-n systems, one can see Li and Zhao (2006, 2008), Hu et al. (2007), Navarro and Eryilmaz (2007), Bairamov and Arnold (2008), and Navarro et al. (2008a).

Khaledi and Shaked (2007) and Li and Zhang (2008) subsequently extended some ordering properties of the residual lives and inactivity times of some particular coherent systems with independent and identically distributed (i.i.d.) components to general coherent systems with i.i.d. ones by using the concept of signatures.

Dependence among component lifetimes is from the common random production and operating environments. It might be difficult to compute and analyze reliability characteristics of a system consisting of dependent components. The exchangeability of components means that they have identical distributions, but the failure of a component affects the other ones in the system. Recent discussions on systems with exchangeable components appear in the works of Navarro and Hernandez (2008), Navarro et al. (2008b), and Navarro (2008).

This article investigates some ordering properties of the residual lives and inactivity times of general coherent systems with exchangeable components by using signatures, extending the case that components are i.i.d. In Section 2, some definitions and notations closely related to the main conclusions are introduced. In Sections 3 and 4, some stochastic comparisons of residual lives, and inactivity times, respectively, between two systems with respect to some orders are obtained. These results are an almost immediate extension of the results in Li and Zhang (2008).

2. Notations and definitions

In this section, we recall several criteria to compare random variables, the concept of signature of a coherent system, all of them are closely related to those main results to be presented in this paper.

Let X and Y be the lifetimes of two components, with their distribution functions $F(x)$ and $G(x)$, and their probability density functions $f(x)$ and $g(x)$, respectively. Denote their survival functions by $\bar{F}(x)=1-F(x)$ and $\bar{G}(x)=1-G(x)$, respectively.

Definition 1. X is said to be smaller than Y in the

(a) Usual stochastic order (denoted by $X\leq_{st}Y$) if $\bar{F}(x)\leq\bar{G}(x)$ for all x.
(b) Hazard rate order (denoted by $X\leq_{hr}Y$) if $\bar{F}(x)/\bar{G}(x)$ is *decreasing* in x.
(c) Reversed hazard rate order (denoted by $X\leq_{rh}Y$) if $F(x)/G(x)$ is *decreasing* in x.
(d) Likelihood ratio order (denoted by $X\leq_{lr}Y$) if $f(x)/g(x)$ is *decreasing* in x in the union of their *supports*.

Definition 2. For two discrete distributions $\mathbf{p}=(p_1,\ldots,p_n)$ and $\mathbf{q}=(q_1,\ldots,q_n)$, $\mathbf{p}$ is said to be smaller than $\mathbf{q}$ in the

(a) Usual stochastic order (denoted by $\mathbf{p}\leq_{st}\mathbf{q}$) if $\sum_{j=i}^n p_j\leq\sum_{j=i}^n q_j$ for all $i=1,2,\ldots,n$.
(b) Hazard rate order (denoted by $\mathbf{p}\leq_{hr}\mathbf{q}$) if $\sum_{j=i}^n p_j/\sum_{j=i}^n q_j$ is *decreasing* in i.
(c) Reversed hazard rate order (denoted by $\mathbf{p}\leq_{rh}\mathbf{q}$) if $\sum_{j=1}^i p_j/\sum_{j=1}^i q_j$ is *decreasing* in i.
(d) Likelihood ratio order (denoted by $\mathbf{p}\leq_{lr}\mathbf{q}$) if p_i/q_i is *decreasing* in i, when $p_i,q_i>0$.

It is well-known that

$$X\leq_{lr}Y \implies X\leq_{hr}(\leq_{rh})Y \implies X\leq_{st}Y. \tag{1}$$

For more comprehensive discussions on properties, and other details of those stochastic orderings, we refer the reader to Shaked and Shanthikumar (2007).

Let $X_1,\ldots,X_n$ be lifetimes of n i.i.d. components of a coherent system. Denote $X_{i,n}$ the i-th smallest order statistics among $X_1,\ldots,X_n$. For a coherent system with n i.i.d. lifetimes $X_1,\ldots,X_n$, its lifetime can be expressed as $T=\tau(X_1,\ldots,X_n)$, where τ is a coherent life function (see Esary and Marshall, 1970; Barlow and Proschan, 1981 for the definitions and properties of coherent life functions). Samaniego (1985) firstly defined the signature of a coherent system as a probability vector $\mathbf{p}=(p_1,\ldots,p_n)$ with

$$\begin{aligned} p_i &= P(\tau(X_1,\ldots,X_n)=X_{i,n}) \\ &= \frac{\text{number of orderings for which the } i\text{-th failure causes the system failure}}{n!}, \end{aligned}$$

$\sum_{i=1}^n p_i=1$. Also Samaniego (1985) and Kochar et al. (1999) subsequently showed that a coherent system having i.i.d. units is a mixture of k-out-of-n system with weights p_i. Navarro et al. (2005) observed it holds when the components of the system are absolutely continuous and exchangeable (i.e. the joint survival function $R(x_1,\ldots,x_n)$ of $X_1,\ldots,X_n$ is symmetric in $x_1,\ldots,x_n$). Recently, Navarro et al. (2008a, b) yielded a closed-form expression for the signature of a system with n i.i.d. components, which is equivalent (equivalent means the same distributions of the lifetimes of the systems) to an arbitrary mixed system of order $k<n$, where a mixed system refers to stochastic mixtures of coherent systems of a given size, according to a fixed and known probability distribution. This made it possible to compare systems of different sizes.

Khaledi and Shaked (2007) further pointed out that the signature of a coherent system, which functions with probability 1 as long as at least $n-s+1$ $(1\leq s\leq n)$ components function, must be of the form $(0,\ldots,0,p_s,\ldots,p_n)$. It was also showed there that