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Empirical likelihood for quantiles under negatively associated samples

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ABSTRACT

In this paper, we study the construction of confidence intervals for quantiles of a population under a negatively associated sample by using the blockwise technique. It is shown that the blockwise empirical likelihood (EL) ratio statistic is asymptotically χ^2-type distributed. The result is used to obtain EL based confidence intervals for quantiles of a population. Results of a simulation study on the finite sample performance of the proposed confidence intervals are reported.

1. Introduction

Random variables $\{\eta_i, 1 \le i \le n\}$ are said to be negatively associated (NA), if for every pair of disjoint subsets A_1, A_2 of $\{1, 2, \ldots, n\}$ and any real-valued coordinatewise increasing functions f and g,

$$Cov\{f(\eta_i, i \in A_1), g(\eta_j, j \in A_2)\} \le 0,$$

provided $Ef^2(\eta_i, i \in A_1) < \infty$, $Eg^2(\eta_j, j \in A_2) < \infty$.

The concept of negatively associated random variables was introduced and studied by Block et al. (1982) and Joag-Dev and Proschan (1983). As pointed out and proved by Joag-Dev and Proschan (1983), a number of well known multivariate distributions possess the NA property, such as (a) multinomial, (b) convolution of unlike multinomials, (c) multivariate hypergeometric, (d) Dirichlet, (e) Dirichlet compound multinomial, (f) negatively correlated normal distribution, (g) permutation distribution, (h) random sampling without replacement, and (i) joint distribution of ranks. The convergence of the sums of negatively associated random variables, because of their wide applications, was studied extensively by Su et al. (1997), Huang and Xu (2002), Matula (1992), Liang and Su (1999), Shao (2000), Zhang (2001) and Li and Zhang (2004), among others.

Let $X_1, X_2, \ldots, X_n$ be a sequence of negatively associated random variables, and F be the distribution function of their common population. The aim of this paper is to construct confidence intervals for $\theta_\gamma =: F^{-1}(\gamma) = \inf\{x|F(x) \ge \gamma\}$, the γ-th quantile of F for $\gamma \in (0,1)$. In financial mathematics and financial risk management, θ_γ is called the value-at-risk (VaR) which specifies the level of excessive losses at a confidence level $1-\gamma$. VaR is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. Cai and Roussas (1997) studied the asymptotic normality of a smooth estimator of θ_γ under association and other dependent samples. However, a variance estimator is required to obtain a confidence interval for θ_γ based on the normal approximation. This estimation of variance is much more involved than independent cases. Our aim is to

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construct the empirical likelihood (EL) confidence intervals for θ_γ with a stationary negatively associated sample by using the blockwise technique without a secondary variance estimation.

The EL method to construct confidence intervals, proposed by Owen (1988, 1990), has many advantages over its counterparts like the normal-approximation-based method and the bootstrap method (e.g., Hall and La Scala, 1990; Hall, 1992). Two features of EL confidence intervals are worth notice here. Their shape and orientations are naturally determined by data, and the intervals are obtained without a secondary estimation. Chen and Hall (1993) developed the EL method in construction of confidence intervals for quantiles under independent sample. Jing et al. (2009) proposed a jackknife EL method for dealing with non-linear functionals with a particular application to U-statistics. Quantile estimation using EL in the context of survey sampling was studied in Chen and Wu (2002). Kitamura (1997) first proposed blockwise EL method to construct confidence intervals for parameters with mixing samples. Chen and Wong (2009) developed blockwise EL method to construct confidence intervals for quantiles with mixing samples. Blockwise method has also been used in the context of bootstrap approximation (e.g., Shi, 1986) for m-dependent variables.

The rest of the paper is organized as follows. The main results of this paper are presented in Section 2. Results of a simulation study on the finite sample performance of the proposed confidence intervals are reported in Section 3. Some lemmas too prove the main results are given in Section 4. The proofs of the main results is presented in Section 5.

2. Main results

A smooth kernel estimator for $F(x)$ at a given $x \in R$ is

$$F_n(x) = n^{-1}\sum_{i=1}^{n} G_h(x-X_i), \tag{2.1}$$

where $G_h(x) = \int_{-\infty}^{x/h} K(y)\,dy$, K is a kernel function and $h=h_n$ are bandwidths. To obtain the EL confidence intervals on $\theta_\gamma = F^{-1}(\gamma)$, we employ the small-block and large-block arguments for the score functions $G_h(\theta_\gamma - X_i)-\gamma$, $1 \le i \le n$, where the score functions are proposed in Chen and Hall (1993), as follows. Let $e_i = G_h(\theta_\gamma - X_i)-\gamma$, $1\le i \le n$, $k=[n/(p+q)]$, where $[t]$ denotes the integral part of t, and $p=p(n)$ and $q=q(n)$ are positive integers satisfying $p+q \le n$. Put

$$\omega_{n,2m-1}(\theta_\gamma) = \sum_{i=r_m}^{r_m+p-1} e_i, \omega_{n,2m}(\theta_\gamma) = \sum_{i=l_m}^{l_m+q-1} e_i, \omega_{n,2k+1}(\theta_\gamma) = \sum_{i=k(p+q)+1}^{n} e_i,$$

where $r_m=(m-1)(p+q)+1$, $l_m=(m-1)(p+q)+p+1$, $m=1, \dots, k$.

We define the following blockwise EL ratio:

$$R(\theta_\gamma) = \sup_{\tilde{p}_1,\dots,\tilde{p}_{2k+1}} \left\{ \prod_{j=1}^{2k+1} (2k+1)\tilde{p}_j \,\middle|\, \sum_{j=1}^{2k+1} \tilde{p}_j = 1,\ \tilde{p}_j \ge 0,\ \sum_{j=1}^{2k+1} \tilde{p}_j \omega_{nj}(\theta_\gamma) = 0 \right\}.$$

It is easy to obtain the $(-2\log)$ blockwise EL ratio statistic

$$\ell(\theta_\gamma) = 2\sum_{j=1}^{2k+1} \log\{1+\lambda(\theta_\gamma)\omega_{nj}(\theta_\gamma)\}, \tag{2.2}$$

where $\lambda(\theta_\gamma)$ is determined by

$$\sum_{j=1}^{2k+1} \frac{\omega_{nj}(\theta_\gamma)}{1+\lambda(\theta_\gamma)\omega_{nj}(\theta_\gamma)} = 0. \tag{2.3}$$

To obtain the asymptotical distribution of $\ell(\theta_\gamma)$, we need the following assumptions.

Assumptions. (A1) (i) The $X_1, X_2, \dots, X_n$ form a stationary sequence of real-valued r.v.s. with distribution function F and bounded probability density function f.

(ii) The X_i's are negatively associated with $EX_i^2 < \infty$.

(iii) Let $u(n) = \sum_{j=n}^{\infty} |Cov(X_1, X_{j+1})|^{1/3}$ and suppose that $u(1) < \infty$.

(iv) The derivative $f'(x)$ exists for $x \in R$ and is bounded on R.

(A2) The function K is a probability density function and satisfies

$$\int_R uK(u)\,du = 0, \quad \int_R u^2 K(u)\,du < \infty.$$

(A3) The sequence of bandwidths $\{h = h_n, n \ge 1\}$ satisfies

$$0 < h \to 0, \quad nh \to \infty, \quad nh^4 \to 0.$$

(A4) Let p, q and k be as described above, which satisfy (i) $q \to \infty$ and $q/p \to 0$. (ii) $p^2/n \to 0$.