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Selection between models through multi-step-ahead forecasting

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ABSTRACT

We develop and show applications of two new test statistics for deciding if one ARIMA model provides significantly better h-step-ahead forecasts than another, as measured by the difference of approximations to their asymptotic mean square forecast errors. The two statistics differ in the variance estimates used for normalization. Both variance estimates are consistent even when the models considered are incorrect. Our main variance estimate is further distinguished by accounting for parameter estimation, while the simpler variance estimate treats parameters as fixed. Their broad consistency properties offer improvements to what are known as tests of Diebold and Mariano (1995) type, which are tests that treat parameters as fixed and use variance estimates that are generally not consistent in our context. We show how these statistics can be calculated for any pair of ARIMA models with the same differencing operator.

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1. Introduction

In this article, we make several contributions to the technology of testing whether two not necessarily correct time series models for an observed series have equal or differing h-step-ahead forecasting ability as assessed by estimates of mean square h-step forecast error. This work is in the tradition of Meese and Rogoff (1988), Findley (1990, 1991a), Diebold and Mariano (1995) and Rivers and Vuong (2002). Our focus is on nonstationary ARIMA models, a type of model not considered in this earlier work. Our specific approach is derived from the goodness-of-fit testing methodology of McElroy and Holan (2009) with modifications to account for the consideration of more than one model and other features of the forecast comparison setting. We account for effects of parameter estimation, which only Rivers and Vuong (2002) do among the forecasting papers cited. In contrast to Rivers and Vuong, we provide explicit formulas for the asymptotic variance of our statistic (corresponding to the σ_n^2 quantity of their Assumption 7), as well as an explicit consistent estimator of this variance. Also, our assumptions are more basic and therefore more transparent. These same advantages apply in relation to the results of West (1996), which also account for parameter estimation but are focused on out-of-sample forecasting, from a perspective more connected with regression models. Our tests, like those of the papers other than West's, are tests of in-sample forecast performance.

The approximation relation between our measure of model forecast performance (8) and the more customary average of squared forecast errors over the sample is derived in Section 2.1, after a review of some relevant aspects of ARIMA model forecasting. The central theoretical results of the paper are presented in Section 2.3, whose Theorem 1 provides the CLT and consistent estimator of its variance needed for our main test statistic (12). Section 2.4 presents results for the situation in which parameter estimation uncertainty is ignored, i.e., when estimated parameters are treated as constant. Here our consistent variance estimate simplifies, becoming reasonably straightforward to calculate for all ARIMA models, and is also

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applicable to the ARIMA model case of the test commonly referred to as the test of Diebold and Mariano (1995). For this test, it provides a consistent alternative to the customary variance estimate, which is consistent only in effectively correct model situations. With $h=1$, it also provides a consistent variance estimate, which had been lacking, for the time series generalization in Findley (1990) of the nonnested model comparison test statistic of Vuong (1989).

In Section 3, after explaining why the size study of Diebold and Mariano (1995) is invalid, we present size and power studies of our test statistics and the Diebold–Mariano statistic together with an empirical study of the application of all three statistics to competing models for series from Box et al. (1994) and Brockwell and Davis (2002). The size and power studies favor both of our new test statistics over the Diebold–Mariano statistic. All of the studies favor most our statistic that accounts for parameter estimation.

The Appendix contains proofs and the derivations of some formulas, including auxiliary formulas for algebraically computing the variance estimate that accounts for parameter uncertainty.

2. Methodology

We are interested in comparing two competing models' h-step-ahead forecasts of data from a time series Y_t which, if nonstationary, can be made stationary by application of a differencing operator, i.e., a backshift operator polynomial $\delta(B)$ whose zeroes have unit magnitude. As usual, B denotes the backshift (or lag) operator, with $BX_t = X_{t-1}$. To simplify the exposition, the stationary series $W_t = \delta(B)Y_t$ is assumed to be Gaussian, an assumption that can be weakened moderately. It is also assumed to be purely nondeterministic. Thus its spectral density $\tilde{f}$ is log integrable and generates its autocovariances via $\gamma_j(\tilde{f}) = (2\pi)^{-1}\int_{-\pi}^{\pi}\tilde{f}(\lambda)e^{ij\lambda}\,d\lambda$, a formula that shows our convention with the constant 2π. The matrix of autocovariances is denoted $\Gamma(\tilde{f})$, i.e., $\Gamma_{jk}(\tilde{f}) = \gamma_{j-k}(\tilde{f})$. The dimension of $\Gamma(\tilde{f})$ is equal to the number of $W_t = \delta(B)Y_t$ calculable from the observed Y_t.

2.1. Multi-step-ahead forecasting

We start by reviewing some basic forecasting results for nonstationary Y_t. Beyond basic formulas, the key results obtained are two concerning asymptotic properties of forecast error measures, (6) and (7).

Let $\delta(z) = 1 + \sum_{j=1}^{d}\delta_j z^j$ be the differencing operator such that $W_t = \delta(B)Y_t$ and let Y_t, $1-d \le t \le n$ denote the available data. Set $\tau(z) = 1/\delta(z)$ expressed as a power series in $|z|<1$ with coefficients τ_j. Thus $\tau_j = 1$ for $j=0$ and $\tau_j = -\sum_{i=0}^{j-1}\tau_i\delta_{j-i}$ for $j>0$. For any $1 \le h < n$ and any $1 \le t \le n-h$, we have $Y_{t+h} = [\tau]_0^{h-1}(B)W_{t+h} + \sum_{j=0}^{d-1} c_{j,h}Y_{t-j}$, where the coefficients $c_{j,h}$ depend only on the coefficients of $\delta(z)$, see Bell (1984, p. 650). The bracket notation means that the power series is truncated to powers of B between zero and $h-1$. Forecasts $\hat{Y}_{t+h|t}$ of Y_{t+h} from Y_s, $1-d \le s \le t$ are obtained from forecasts $\hat{W}_{t+h-j|t}$, $0 \le j \le h-1$ of W_{t+h-j} from W_s, $1 \le s \le t$ by way of

$$\hat{Y}_{t+h|t} = \sum_{j=0}^{h-1}\tau_j\hat{W}_{t+h-j|t} + \sum_{j=0}^{d-1}c_{j,h}Y_{t-j}. \tag{1}$$

Consequently, the forecast errors are given by $Y_{t+h} - \hat{Y}_{t+h|t} = \sum_{j=0}^{h-1}\tau_j(W_{t+h-j} - \hat{W}_{t+h-j|t})$.

To motivate our performance measure, we will use the forecast $\hat{W}_{t+h|t}$ obtained by truncating the filter for the forecast $W_{t+h|t}$ of W_{t+h} from the infinite past W_s, $-\infty < s \le t$. The latter forecast is given by $W_{t+h|t} = \sum_{j\ge 0}\psi_{j+h}B^j\Psi(B)^{-1}W_t$, where $\Psi(z) = \sum_{j\ge 0}\psi_j z^j$ with $\psi_0 = 1$ has the coefficients of the innovations (Wold, MA(∞)) representation $W_t = \sum_{j\ge 0}\psi_j\varepsilon_{t-j}$ with ε_t the error of the mean square optimal forecast of W_t from W_s, $s<t$. Since $W_{t+h} - W_{t+h|t} = [\Psi]_0^{h-1}(B)\Psi^{-1}(B)W_{t+h} = [\Psi]_0^{h-1}(B)\varepsilon_{t+h}$, this forecast error is a moving average process of order (at most) $h-1$, as is also the error process of the forecasts $Y_{t+h|t} = \sum_{j=0}^{h-1}\tau_j W_{t+h-j|t} + \sum_{j=0}^{d-1}c_{j,h}Y_{t-j}$,

$$Y_{t+h} - Y_{t+h|t} = \sum_{j=0}^{h-1}\tau_j(W_{t+h-j} - W_{t+h-j|t}) = \sum_{j=0}^{h-1}\tau_j B^j[\Psi]_0^{h-1-j}(B)\Psi^{-1}(B)W_{t+h} = \sum_{j=0}^{h-1}\tau_j B^j[\Psi]_0^{h-1-j}(B)\varepsilon_{t+h}, \tag{2}$$

where the backshift operators B^j operate on the t index.

The truncated filter forecast $\hat{W}_{t+h|t}$ and its error $W_{t+h} - \hat{W}_{t+h|t}$ are obtained from the infinite past formulas given above by setting $W_{t-j}=0$ for $j \ge t$. Denoting the filter in (2) by

$$\eta^{(h)}(B) = \left(\sum_{j=0}^{h-1}\tau_j B^j[\Psi]_0^{h-1-j}(B)\right)\Psi^{-1}(B), \tag{3}$$

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