Journal of Statistical Planning and Inference 139 (2009) 2715–2730

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

Testing of a sub-hypothesis in linear regression models with long memory errors and deterministic design

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ARTICLE INFO

Article history:
Received 28 May 2008
Received in revised form
21 November 2008
Accepted 19 December 2008
Available online 7 January 2009

MSC:
primary 62M09
secondary 62M10, 62M99

Keywords:
Moving averages
Whittle quadratic forms

ABSTRACT

This paper considers the problem of testing a sub-hypothesis in homoscedastic linear regression models where errors form long memory moving average processes and designs are non-random. Unlike in the random design case, asymptotic null distribution of the likelihood ratio type test based on the Whittle quadratic form is shown to be non-standard and non-chi-square. Moreover, the rate of consistency of the minimum Whittle dispersion estimator of the slope parameter vector is shown to be $n^{-(1-\alpha)/2}$, different from the rate $n^{-1/2}$ obtained in the random design case, where α is the rate at which the error spectral density explodes at the origin. The proposed test is shown to be consistent against fixed alternatives and has non-trivial asymptotic power against local alternatives that converge to null hypothesis at the rate $n^{-(1-\alpha)/2}$.

1. Introduction

When fitting a polynomial regression model, a relevant question to ask is what is the degree of the polynomial being fitted to the given data. One way to address this question is to first assume one has a polynomial regression model of a given degree and then to test for the sub-hypothesis that certain higher order coefficients are vanishing.

More generally, let n, p and k be known positive integers with $k \leqslant p$. Let $\boldsymbol{X}_{t1}, 1 \leqslant t \leqslant n$ and $\boldsymbol{X}_{t2}, 1 \leqslant t \leqslant n$ be, respectively, $k \times 1$ and $(p-k) \times 1$ real vectors and Y_t, $1 \leqslant t \leqslant n$ denote the response variables. Consider the regression model where for some $\beta_1 \in \mathbb{R}^k$ and $\beta_2 \in \mathbb{R}^{p-k}$,

$$Y_t = \beta_1' \boldsymbol{X}_{t1} + \beta_2' \boldsymbol{X}_{t2} + \varepsilon_t, \quad t = 1, \ldots, n. \tag{1.1}$$

The problem of interest is to test

$$H_0 : \beta_2 = 0 \quad \text{vs.} \quad H_1 : \beta_2 \neq 0.$$

In the case of independent homoscedastic errors $\{\varepsilon_t\}$ and when the design variables are either non-random or random and i.i.d., this problem is well studied in the literature, cf. Rao (1973) and Hart (1997) and references therein. A classical testing procedure

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[1] Research of this author partly supported by the NSF DMS Grant 0701430.
[2] Research of this author partly supported by the Bilateral France–Lithuania Scientific Project Gilibert and the Lithuanian State Science and Studies Foundation Grant T-25/08.

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doi:10.1016/j.jspi.2008.12.011

is the likelihood ratio test when errors are Gaussian or the analysis of variance type tests via the least square theory which are asymptotically valid without the Gaussianity assumption.

The focus of this paper is to obtain asymptotic null distribution of an analog of the likelihood ratio type test for H_0 when errors form a long memory moving average process. A discrete time strictly stationary stochastic process with finite second moment is said to have long memory if its auto-covariances tend to zero at a hyperbolic rate as the lag tend to infinity but their sum diverges. Long memory processes arise in numerous physical and social sciences. See Beran (1992, 1994), Baillie (1996), Dehling et al. (2002) and Doukhan et al. (2003) for more on these processes and their numerous applications. Regression models with long memory errors are useful in economics and finance, see e.g., Yajima (1991), Robinson and Hidalgo (1997), Koul et al. (2004), Guo and Koul (2007) among others.

Let $\mathbb{Z} := \{0, \pm 1, \ldots\}$ and Θ be an open and relatively compact subset of $\mathbb{R}^d$, $d \geqslant 1$. The process $\{\varepsilon_t, t \in \mathbb{Z}\}$ is said to form a long memory moving average if for some functions α from Θ to $(0,1)$, a from $\mathbb{Z} \times \Theta$ to $\mathbb{R}$ and c from Θ to $\mathbb{R}$,

$$\varepsilon_t = \sum_{s \in \mathbb{Z}} a(t-s;\vartheta)\zeta_s, \quad t \in \mathbb{Z}, \tag{1.2}$$

where $a(t;\vartheta) \sim c(\vartheta)t^{\alpha(\vartheta)/2-1}$ $(t \to \infty)$ and ζ_j, $j \in \mathbb{Z}$, are standardized i.i.d random variables having finite fourth moment. Let $\hat{a}(u;\vartheta) = (2\pi)^{-1}\sum_{t\in\mathbb{Z}} a(t;\vartheta)\mathrm{e}^{-\mathbf{i}tu}$, $u \in \Pi := [-\pi,\pi]$, be the Fourier transform of a, where $\mathbf{i} = (-1)^{1/2}$. Then the corresponding spectral density is $f(u;\vartheta) = 2\pi|\hat{a}(u;\vartheta)|^2$, $\vartheta \in \Theta$, $u \in \Pi$. Let

$$\hat{b}(u;\vartheta) = \frac{1}{f(u;\vartheta)}, \quad u \in \Pi; \quad b_t(\vartheta) = \int_{\Pi} \mathrm{e}^{\mathbf{i}\,tu}\hat{b}(u;\vartheta)\,\mathrm{d}u, \quad t \in \mathbb{Z}, \quad \vartheta \in \Theta.$$

Let $\boldsymbol{X}_t' := (\boldsymbol{X}_{t1}', \boldsymbol{X}_{t2}')$, $\beta' := (\beta_1', \beta_2')$ and define

$$\Lambda_n(\vartheta,\beta) := \sum_{t,s=1}^{n} b_{t-s}(\vartheta)(Y_t - \boldsymbol{X}_t'\beta)(Y_s - \boldsymbol{X}_s'\beta), \quad \beta \in \mathbb{R}^p,$$

$$(\theta_n, \beta_n) := \operatorname*{argmin}_{(\vartheta,\beta)\in\Theta\times\mathbb{R}^p} \Lambda_n(\vartheta,\beta),$$

$$\Lambda_{n1}(\vartheta,\beta_1) := \sum_{t,s=1}^{n} b_{t-s}(\vartheta)(Y_t - \boldsymbol{X}_{t1}'\beta_1)(Y_s - \boldsymbol{X}_{s1}'\beta_1), \quad \beta_1 \in \mathbb{R}^k, \ \vartheta \in \Theta,$$

$$(\theta_{n1}, \beta_{n1}) := \operatorname*{argmin}_{(\vartheta,\beta_1)\in\Theta\times\mathbb{R}^k} \Lambda_{n1}(\vartheta,\beta_1).$$

Analog of the likelihood ratio test for H_0 would be based on

$$\mathcal{Q}_n := -2[\Lambda_n(\theta_n,\beta_n) - \Lambda_{n1}(\theta_{n1},\beta_{n1})].$$

Recall that in the case of Gaussian errors the exact likelihood ratio test would have the elements of the inverse of the covariance matrix as weights in the quadratic forms Λ_n's instead of $\{b_{t-s}(\vartheta)\}$. The above quadratic forms are their Whittle approximations. For that reason we shall call the test that rejects H_0 in favor of H_1 when $\mathcal{Q}_n$ is large the Whittle test.

In the next section we shall give some sufficient conditions on the design $\{\boldsymbol{X}_t\}$ and on the spectral density f under which the null distribution of $\mathcal{Q}_n$ is seen to converge to a weighted chi-square distribution. This limiting null distribution of $\mathcal{Q}_n$ is different from what is available in the case of random design. Koul and Surgailis (2008) proved that if the design process $\{\boldsymbol{X}_t, t \in \mathbb{Z}\}$ is random having short or long memory, mean zero and is independent of $\{\varepsilon_t, t \in \mathbb{Z}\}$, then $\mathcal{Q}_n$ tends weakly to a chi-square r.v. The conditions on f in Koul and Surgailis (2008) are similar as given below.

The main proofs appear in Sections 3–5. Section 6 gives an approximation of $\mathcal{Q}_n$ under alternatives. It is useful for proving consistency of the Whittle test against fixed alternatives and for discussing some aspects of its asymptotic power against certain sequences of local alternatives. In particular it is observed that the Whittle test has non-trivial asymptotic power against the alternatives $\beta_2 = n^{-(1-\alpha)/2}v, v \neq 0, v \in \mathbb{R}^{p-k}$. Appendix A contains explicit computations of the quantities characterizing the limiting null distribution of $\mathcal{Q}_n$ in the case of polynomial design.

2. Assumptions and main results

In this section we discuss asymptotic null distribution of $\mathcal{Q}_n$. To proceed we need to describe the additional needed assumptions on the spectral density $f(u;\vartheta)$. Below, θ denotes the true value of ϑ and Θ_0 denotes an arbitrarily small neighborhood of θ. Also, let $\beta_0 := (\beta_{01}', \beta_{02}')'$ denote the true value of the parameter vector β for which (1.1) holds.