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Bayes estimation based on k-record data from a general class of distributions under balanced type loss functions

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ABSTRACT

A semi-parametric class of distributions that includes several well-known lifetime distributions such as exponential, Weibull (one parameter), Pareto, Burr type XII and so on is considered in this paper. Bayes estimation of parameters of interest based on k-record data under balanced type loss functions is developed; and in some cases the admissibility or inadmissibility of the linear estimators is considered. The results are presented under the balanced versions of two well-known loss functions, namely squared error loss (SEL) and Varian's linear-exponential (LINEX) loss. Some recently published results on Bayesian estimation using record data are shown to be special cases of our results.

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1. Introduction

1.1. Record data

Let $\{X_i, i \geqslant 1\}$ be a sequence of iid absolutely continuous random variables distributed according to the cumulative distribution function (cdf) $F(\cdot;\theta)$ and probability density function (pdf) $f(\cdot;\theta)$, where θ is an unknown parameter. An observation X_j is called an *upper record value* if its value exceeds all previous observations. Thus, X_j is an *upper record* if $X_j > X_i$ for every $i < j$. Analogously, an upper k-record value is defined in terms of the k-th largest X yet seen. It is of interest to note that there are situations in which only records are observed, such as in destructive stress testing, meteorology, hydrology, seismology, and mining. For a more specific example, consider the situation of testing the breaking strength of wooden beams as described in Glick (1978). Interest in records has increased steadily over the years since Chandler's (1952) formulation. Useful surveys are given by the books of Arnold et al. (1998), Nevzorov (2001) and the references therein. For a formal definition of k-records, we consider the definition

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in Arnold et al. (1998, p. 43) for the continuous case. Let $T_{1,k} = k$ and, for $n \geqslant 2$,

$$T_{n,k} = \min\{j : j > T_{n-1,k}, X_j > X_{T_{n-1,k}-k+1:T_{n-1,k}}\},$$

where $X_{i:m}$ denotes the i-th order statistic in a sample of size m. The sequences of *upper k-records* are then defined by $R_{n(k)} = X_{T_{n,k}-k+1:T_{n,k}}$ for $n \geqslant 1$. For $k = 1$, note that the usual records are recovered. These sequences of k-records were introduced by Dziubdziela and Kopocinski (1976) and they have found acceptance in the literature. Nagaraja (1988) pointed out that k-records with an underlying distribution function F can be viewed as ordinary record values ($k = 1$) based on the distribution of the minimum with cdf $F_{1:k} \equiv 1 - \{1 - F\}^k$. Using the joint density of usual records, the marginal density of $R_{n(k)}$ is obtained as

$$f_{n,k}(r_{n(k)}; \theta) = \frac{k^n}{(n-1)!}[-\log \bar{F}(r_{n(k)}; \theta)]^{n-1}\{\bar{F}(r_{n(k)}; \theta)\}^{k-1} f(r_{n(k)}; \theta), \tag{1}$$

and the joint pdf of the first n k-records is given by

$$f_{1,\ldots,n}(\mathbf{r}; \theta) = k^n [\bar{F}(r_{n(k)}; \theta)]^k \prod_{i=1}^{n} \frac{f(r_{i(k)}; \theta)}{\bar{F}(r_{i(k)}; \theta)}, \tag{2}$$

where $\mathbf{r} = (r_{1(k)}, \ldots, r_{n(k)})$ and $\bar{F} = 1 - F$ (see Arnold et al., 1998). The problem of estimation based on record data has been previously studied in the literature, in particular under a Bayesian framework (e.g., Ali Mousa et al., 2002; Jaheen, 2003, 2004; Ahmadi et al., 2005; Ahmadi and Doostparast, 2006). The goal of this paper is to develop Bayes estimation of functions of θ based on k-record data from a general class of distributions under balanced type loss functions, which we now describe.

1.2. Balanced type loss functions

To reflect both goodness of fit and precision of estimation in estimating an unknown parameter θ under the model $\mathbf{X} = (X_1, \ldots, X_n) \sim F_\theta$, Zellner (1994) introduced a balanced loss function (BLF) as follows:

$$\frac{\omega}{n} \sum_{i=1}^{n} (X_i - \delta)^2 + (1 - \omega)(\delta - \theta)^2,$$

where $\omega \in [0, 1)$, and considered optimal estimates relative to BLF for estimation of a scalar mean, a vector mean and a vector regression coefficients. Dey et al. (1999), as well as Jafari Jozani et al. (2006a) studied the notion of a BLF from the perspective of unifying a variety of results both frequentist and Bayesian. They showed in broad generality that frequentist and Bayesian results for BLF follow from and also imply related results for SEL functions. Jafari Jozani et al. (2006b) introduced an extended class of balanced type loss functions of the form

$$L^q_{\rho,\omega,\delta_0}(\gamma(\theta), \delta) = \omega q(\theta) \rho(\delta_0, \delta) + (1 - \omega) q(\theta) \rho(\gamma(\theta), \delta), \tag{3}$$

with $q(\cdot)$ being a suitable positive weight function, $\rho(\gamma(\theta), \delta)$ being as arbitrary loss function in estimating $\gamma(\theta)$ by δ, and δ_0 a chosen a priori "target" estimator of $\gamma(\theta)$, obtained for instance from the criterion of maximum likelihood, least squares or unbiasedness among others. They give a general Bayesian connection between the cases $\omega > 0$ and $\omega = 0$. For the case of squared error ρ, a least squares δ_0, $\gamma(\theta) = \theta$, and $q(\theta) = 1$ in (3) is equivalent to Zellner's (1994) BLF, and the introduction of an arbitrary ρ extends the squared error version of (3) introduced by Jafari Jozani et al. (2006a). In this paper, we shall use balanced squared error loss (balanced SEL) and balanced LINEX loss to illustrate Bayesian estimation of parameters of interest in a class of distributions (described in Section 1.3) based on a sample of k-record values. A companion paper (Ahmadi et al., 2008) considers a similar framework but for prediction of future k-records. Much of the analysis below is unified with respect to the choice of the target estimator δ_0, the weight ω, and to some extent, with respect to the parameter being estimated.

1.3. Proportional hazard rate models

Let $X_1, X_2, \ldots$ be a sequence of iid random variables from the family of continuous distribution functions with

$$F(x; \theta) = 1 - [\bar{G}(x)]^{\alpha(\theta)}, \quad -\infty \leqslant c < x < d \leqslant \infty, \tag{4}$$

where $\alpha(\theta) > 0$, $G \equiv 1 - \bar{G}$, and G is an arbitrary continuous distribution function with $G(c) = 0$ and $G(d) = 1$. The family in (4) is well-known in lifetime experiments as proportional hazard rate models (see for example Lawless, 2003), and includes several well-known lifetime distributions such as exponential, Pareto, Lomax, Burr type XII, and so on.

Let $g(x) = (\mathrm{d}/\mathrm{d}x)G(x)$ be the corresponding pdf, then

$$f(x; \theta) = \alpha(\theta) g(x) [\bar{G}(x)]^{\alpha(\theta)-1}, \quad -\infty \leqslant c < x < d \leqslant \infty. \tag{5}$$

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