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Projection estimation capacity of Hadamard designs

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Abstract

In a screening design, often only a few factors among a large number of potential factors are significantly important. Usually, it is not known which factors will be important ones. Thus, it is of practical interest to know if each projection of a design onto a small subset of factors is able to entertain and estimate all two-factor-interactions along with its main effects, assuming higher order interactions are negligible. In this paper, we investigate the estimation capacity of projections of Hadamard designs with run size up to 60. Possible applications of our results to robust parameter designs are also discussed.

MSC: Primary, 62K15; secondary 05B20

Keywords: Estimation capacity; Hadamard design; Projection; Screening design

1. Introduction

An $n \times n$ matrix H_n with entries $+1$ and -1 is called a Hadamard matrix of order n if any two columns (and hence any two rows) of H_n are orthogonal. Such a matrix can be normalized so that all entries in the first column are equal to 1. After deleting the first column, the resulting design is often referred to a Hadamard design, and it can be used to conduct an experiment with $n-1$ factors in n runs. The classical Plackett–Burman designs from Plackett and Burman (1946) are special cases of Hadamard designs. Due to their orthogonality and economy in run sizes, Hadamard designs are an important source of screening designs. At the preliminary stage of an experimental investigation, it is believed that only a few out of a large number of potential factors are important factors. This principle is referred to *factor sparsity*. Often, it is not known which a few factors will be important. Thus, it is important to study all projections of a Hadamard design onto a small subset of factors.

Designs constructed using Hadamard matrices, including Plackett–Burman designs and Hadamard designs, are an important class of two-level orthogonal arrays, which have been extensively used in factorial designs. Specifically, a two-level orthogonal array of size n, with m constraints and strength t, denoted by $\mathrm{OA}(n, 2^m, t)$, is an $n \times m$ matrix with entries $+1$ and -1 such that the 2^t possible level combinations in any submatrix with t columns occur equally frequently. Such an array can be regarded as a two-level factorial design of n runs and m factors with a property that with regard to any t factors the design is a replicated full 2^t factorial. Hadamard designs regarded as orthogonal arrays often are of strength $t = 2$. Box and Tyssedal (1996) introduced the concept of projectivity as an extension of the

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concept of strength of an orthogonal array. An $n \times m$ design D with n observations and m factors each at two levels is said to be of projectivity p if each projection of design D onto any p out of m factors contains a full 2^p factorial design (possibly with some runs replicated). Due to its orthogonality, any Hadamard design has projectivity $p \geqslant 2$. When there are only p or less than p important factors, then the projection onto the important factors of a design with projectivity p guarantees the estimation of all factorial effects. However, the projectivity of a Hadamard design is of $p = 2$ or 3, and the projectivity itself does not provide information in regard to the estimability of factorial effects of projections with more than p factors.

Because of the assumption of factor sparsity, it is important to study projections of a design onto lower dimension in the process of selecting a screening design. Box and Hunter (1961) first discussed projection properties of regular factorial designs through the concept of *resolution*. For nonregular factorial designs (which are often constructed from Hadamard matrices), or more generally, orthogonal arrays, their projection properties have recently been studied extensively, see, for example, Lin and Draper (1992), Cheng (1995), Wang and Wu (1995), Deng and Tang (1999, 2002), Tang (2001) and Xu and Deng (2005). In contrast to the geometric projection aspect (number of runs short from a design with desirable resolution), the more important projection aspect of a design is its ability to estimate interactions in addition to the estimation of all the main effects. It is especially true for nonregular factorial designs as they often have hidden projection property. Cheng (1995) showed that for any two-level orthogonal array with strength $t = 2$, if the run size of the array is not a multiple of 8, then each projection onto any four factors of the array has the ability to entertain estimation of all the main effects and two-factor interactions of these four factors, assuming that the higher order interactions are negligible. In this article, we further investigate hidden projection properties of Hadamard designs and examine their estimation capacity of all projections onto lower dimensions of Hadamard designs with run size up to 60.

One of the most important tools in quality engineering is robust parameter design. Taguchi (1986) introduced this concept with the primary objective of reducing the variation of a product or process by identifying the settings of control factors so that the response variable is insensitive to changes in the noise factors. Russell et al. (2004) provided an algorithm for generating regular fractional factorial designs for this purpose. Our findings of projections of Hadamard designs may provide alternative candidates for robust parameter designs as a single array.

This paper is organized as follows. In Section 2, we introduce the concept of projection estimation capacity and study estimation capacity of projections of Hadamard designs with run sizes up to 60. In Section 3, we discuss the application of the results obtained in Section 2 to robust parameter designs. The conclusion is summarized in Section 4.

2. Projection estimation capacity

A design is said to be of projection estimation capacity (PEC) k if: (1) each projection onto any k factors of the design is able to estimate all the main effects and two-factor interactions of these k factors and (2) there exists at least one projection onto $k + 1$ factors of the design such that this projection does not have the ability to estimate all two-factor interactions along with all the main effects of these $k + 1$ factors. Cheng (1995) and Bulutoglu and Cheng (2003) provided some general theoretical results on projection of nonregular factorial designs. Cheng (1995) showed that as long as the run size n of an OA$(n, 2^m, 2)$ with $m \geqslant 4$ is not a multiple of 8, its projection onto any four columns allows the estimation of all the main effects and two-factor interactions. That is, such an array is of PEC $\geqslant 4$. Bulutoglu and Cheng (2003) further showed that the same desirable hidden projection property also holds for the first type Paley designs (hereafter referred to as Paley designs) with run sizes greater than 8, even when their run sizes are multiples of 8. Paley designs can be obtained by deleting the column of 1's of a normalized Paley matrices, which are a family of Hadamard matrices constructed by Paley (1933). When the run size n is a multiple of 4 and $n - 1$ is an odd prime power, a Paley design can be constructed using Galois field; see, Hedayat et al. (1999) and Bulutoglu and Cheng (2003) for details.

Two Hadamard matrices (or two designs) are said to be equivalent if one can be obtained from the other by permuting rows, columns, switching all the signs in one or more columns, or a combination of the above. It is known that the number of nonequivalent Hadamard matrices of orders 12, 16, 20, 24, and 28 are 1, 5, 3, 60, and 487, respectively; see, Hall (1965) and Spence (1995). However, the complete sets of nonequivalent Hadamard matrices of order 32 or higher are not yet available. Box and Tyssedal (1996) indicated that a nonregular cyclic design is of better projection property due to its cyclic generation property. Therefore, when the complete set of nonequivalent Hadamard designs