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Orthogonal arrays of strength 3 and small run sizes

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Abstract

All mixed (or asymmetric) orthogonal arrays of strength 3 with run size at most 64 are determined.
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1. Introduction

In this paper we study mixed orthogonal arrays of strength 3. Let $s_1, s_2, \ldots, s_k$ be a list of natural numbers, and for each i, let Q_{s_i} be a set of size s_i. For natural numbers t, N, a multiset $\mathscr{F}$ of size N whose elements are from $Q_{s_1} \times Q_{s_2} \times \cdots \times Q_{s_k}$ is called an *orthogonal array of strength t*, notation $OA(N, s_1, s_2, \ldots, s_k, t)$, if $t \leqslant k$, and, for every index set $I \subseteq \{1, \ldots, k\}$ of size at most t, each row of $\prod_{i \in I} Q_{s_i}$ occurs equally often in the projection of $\mathscr{F}$ onto the coordinates indexed by I.

We refer to the elements of $\mathscr{F}$ as *runs*, so N is the number of runs of $\mathscr{F}$, also called its *run size*. The coordinates of $Q_{s_1} \times Q_{s_2} \times \cdots \times Q_{s_k}$ are called *factors*, so k is the number of factors. Moreover, s_i is called the *level* of the factor i. Instead of $s_1, s_2, s_3, \ldots$ we also write $2^a \cdot 3^b \cdot 4^c \ldots$, where the exponents $a, b, c, \ldots$ indicate the number of factors at level 2, 3, 4, etc. An orthogonal array is called *trivial* if it contains each element of $Q_{s_1} \times Q_{s_2} \times \cdots \times Q_{s_k}$ the same number of times.

Orthogonal arrays of strength 2 have been studied extensively. In this paper, we study the case of strength $t = 3$. We restrict ourselves mainly to $N \leqslant 64$.

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Theorem 1. *For every set of parameters* $N, s_1, s_2, \ldots, s_k, t$ *with* $t = 3$ *and* $N \leqslant 64$ *such that an orthogonal array* $OA(N, s_1, s_2, \ldots, s_k, t)$ *exists, we construct at least one such array. More precisely, if* $k = 3$ *such an array is trivial, and if* $k > 3$ *a construction is indicated in Table 1.*

Of course the existence of $OA(N, s_1, s_2, \ldots, s_k, t)$ does not depend on the ordering of the parameters s_j, and we can take them in non-decreasing order if we wish.

Table 1
Parameters of orthogonal arrays of strength 3 with $N \leqslant 64$

N	Levels	Existence	Construction	Nonexistence
8	2^a	$a \leqslant 4$	(H)	
16	$2^a \cdot 4$	$a \leqslant 3$	(M)	
16	2^a	$a \leqslant 8$	(H)	
24	$2^a \cdot 6$	$a \leqslant 3$	(M)	
24	$2^a \cdot 3$	$a \leqslant 4$	(M)	$a = 5$
24	2^a	$a \leqslant 12$	(H)	
27	3^b	$b \leqslant 4$	(L)	$b = 5$
32	$2^a \cdot 8$	$a \leqslant 3$	(M)	
32	$2^a \cdot 4^2$	$a \leqslant 4$	(AD)	
32	$2^a \cdot 4$	$a \leqslant 7$	(M)	
32	2^a	$a \leqslant 16$	(H)	
36	$2^2 \cdot 3^2$		(T)	
40	$2^a \cdot 10$	$a \leqslant 3$	(M)	
40	$2^a \cdot 5$	$a \leqslant 6$	(X_1)	$a = 7$
40	2^a	$a \leqslant 20$	(H)	
48	$2^a \cdot 12$	$a \leqslant 3$	(M)	
48	$2^a \cdot 4 \cdot 6$	$a \leqslant 2$	(M)	$a = 3$
48	$2^a \cdot 6$	$a \leqslant 7$	(M)	
48	$2^a \cdot 3 \cdot 4$	$a \leqslant 4$	(X_2)	$a = 5$
48	$2^a \cdot 4$	$a \leqslant 11$	(M)	
48	$2^a \cdot 3$	$a \leqslant 9$	(X_3)	$a = 10$
48	2^a	$a \leqslant 24$	(H)	
54	$3^b \cdot 6$	$b \leqslant 3$	(M)	$b = 4$
54	$2^a \cdot 3^b$	$a \leqslant 1, b \leqslant 5$	(X_4)	$(a, b) = (0, 6)$
56	$2^a \cdot 14$	$a \leqslant 3$	(M)	
56	$2^a \cdot 7$	$a \leqslant 6$	(J)	$a = 7$
56	2^a	$a \leqslant 28$	(H)	
60	$2^2 \cdot 3 \cdot 5$		(T)	
64	$2^a \cdot 16$	$a \leqslant 3$	(M)	
64	$2^a \cdot 4 \cdot 8$	$a \leqslant 4$	(M)	
64	$2^a \cdot 8$	$a \leqslant 7$	(M)	
64	4^c	$c \leqslant 6$	(L)	
64	$2^a \cdot 4^5$	$a \leqslant 2$	(S)	$a = 3$
64	$2^a \cdot 4^4$	$a \leqslant 6$	(X_5)	
64	$2^a \cdot 4^3$	$a \leqslant 8$	(S)	$a = 9$
64	$2^a \cdot 4^2$	$a \leqslant 12$	(AD)	
64	$2^a \cdot 4$	$a \leqslant 15$	(M)	
64	2^a	$a \leqslant 32$	(H)	

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