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A family of distributions to model load sharing systems

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ABSTRACT

The main characteristic of a load sharing system is that after the failure of one component the surviving components have to shoulder extra load and hence are prone to failure at an earlier time than what is expected under the original model. In others, the failure of one component may release extra resources to the survivors, thus delaying the system failure. In this paper we consider such m component systems and some observation schemes and identifiability issues under them. Then we construct a general semiparametric multivariate family of distributions which explicitly models this phenomenon through proportional conditional hazards. We suggest estimates for the constant of proportionality. We propose a nonparametric test for the hypothesis that the failures take place independently according to the common distribution against the alternative hypothesis that the second failure takes place earlier than warranted, study its properties and illustrate its use.

1. Introduction, literature survey and summary

Let us consider an m component k-out-of-m system. For $1 \leq k \leq m$, the system continues to function as long as $m-k+1$ components have not failed. The series system ($k=m$) and parallel system ($k=1$) are its special cases. However, failure of a component may put additional load on the surviving components and hence affect their functioning and hence the functioning of the system. This may result in stochastic changes in the residual life time of the system. Following examples show that these changes may either decrease or increase (stochastically) the residual lifetime of the system.

Daniels (1945) and Rosen (1964) observed that yarns and cables in a bundle fail only when the last fiber (or wire) in the bundle breaks. A bundle of fibers can be considered as a parallel system subject to a constant tensile load. After a fiber breaks yarn bundles or untwisted cables tend to spread the stress load uniformly on the remaining unbroken fibers. This pioneering work dealt with the strength of the bundles rather than with their lifetimes.

Coleman (1958) obtained the mean time to ultimate failure of a bundle of parallel fibers when the number of fibers becomes large. Birnbaum and Saunders (1958) derived the lifetime distribution of the materials. Phoenix (1978) showed that the system failure is asymptotically normally distributed as the number of components become large.

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Gross et al. (1971) observed that a two organ subsystem (e.g., two kidneys) in a human body typically show this pattern. If a patient gets one of his kidneys removed due to some illness, then the second kidney shows a higher failure rate. The above authors have developed a survival distribution for such two organ systems. Both failure rates are assumed to be constant in time. Iterative estimation procedures for the parameters of the survival distribution are proposed.

Lynch (1999) and Durham and Lynch (2000) studied relationships between the load share rule and the failure rate for some specified load-share rules.

Singpurwalla (1995), Hollander and Pena (1995), Pena (2006) describe these as dynamic reliability systems. Recently Kim and Kvam (2004) have shown the need for such models in several other types of situations.

Cramer and Kamps (1996, 2001) have proposed a new model for the joint distributions involved in the analysis of the k-out-of-m system. The observation scheme is the continuous monitoring scheme, i.e. lifetimes of all the $m-k+1$ components which fail till the system fails are observed sequentially. The joint distribution is specified in terms of the successive conditional distributions of these $m-k+1$ 'sequential order statistics'. Upon a component failure, the working components are assumed to be, conditionally, given the immediately preceding component failure time z, i.i.d. random variables with common cdf

$$\frac{F_i(\cdot)-F_i(z)}{1-F_i(z)}, \quad 1\le i\le m-k+1,$$

where F_i is modelled by $F_i(t)=1-(1-F(t))^{\alpha_i}$. The next component failure time, then, is the minimum of the surviving components. It is clear that given a component failure, this choice will give $\alpha_i f/(1-F)$, that is, α_i times the original hazard rate as the hazard rate of the conditional failure time of a surviving component. Under the i.i.d. set up, without changing the joint distribution in the above manner, this failure rate remains $\alpha_1 f/(1-F)$, that of any component working on its own. Thus the introduction of the unequal parameters α_i may be seen to accommodate load sharing.

However, in this generality, as is the case of dynamic reliability models of Singpurwalla (1995), Hollander and Pena (1995) and Kvam and Pena (2005), it is difficult to obtain workable closed form expressions for the complete joint distributions, or even for the distribution of the system lifetime.

Kim and Kvam (2004) and Kvam and Pena (2005) consider a k component parallel system. Initially the components have identical distribution with failure rate $r(t)$. After the failure of the first component the failure rate of $k-1$ surviving components changes to $\gamma_1 r(t)$, for some $\gamma_1>0$, and so on. They find a nonparametric estimator of the component baseline cumulative hazard function and discuss its asymptotic distribution. They consider the estimation of parameters γ_j's under monotone load sharing subject to the conditions $1\le\gamma_1\le\gamma_2\le\cdots\le\gamma_{k-1}$. They also derive a likelihood ratio test for testing equality of γ's against the alternative that they are monotone.

McCool (2006) modelled the time to failure as a two parameter Weibull distribution. He proposed a test for the hypothesis that the failure of the first component in a parallel system shortens the life of the remaining components of the same system.

In all the above examples the failure of the first component adversely affects the system performance. On the other hand while checking software, detection of a critical fault can help in finding other bugs which are yet undetected. Drummond et al. (2000) carried out a study in a vertebrate species showing that selective deaths due to food shortage result in surviving offspring receiving an increased share of an undiminished food supply. They observed littermates of the domestic rabbit Oryctolagus cuniculus and found that after individual pups died, the total daily milk weight obtained by the litter continued to be the same. The surviving pups showed greater growth as a result of increase in the milk consumption. This necessitates considering the models wherein failure of a component increases the survival chances of the other components.

In this paper we consider k-out-of-m systems. For $k=1$ this reduces to a parallel system. We study their lifelengths, identifiability issues and bounds under some observation schemes in Section 2. In Section 3 we propose a conditional failure rate model which explicitly uses the concept of additional (or decreased) load on the surviving components after the failure of a component. We also discuss the popular Gumbel (1960) and Freund (1961) models from this point of view. Section 4 discusses estimation. Section 5 provides a test for the null hypothesis that a component fails without any additional load on the surviving components against the alternative that there is such an additional load. We also extend the test to the case of right censored data. Section 6 includes a simulation study and in Section 7 the test is illustrated on real data.

2. Identifiability under various observation schemes

Let us denote the lifetimes of the m components by random variables $U_1,U_2,\ldots,U_m$, respectively, and let $X_{(1)},X_{(2)},\ldots,X_{(m)}$ denote the corresponding order statistics. The lifetime of a k-out-of-m system is given by the random variable $Y=X_{(m-k+1)}$. If the labels of the failed components are not recorded then the data will consist of only $X_{(1)},X_{(2)},\ldots,X_{(m-k+1)}$ and the U_i's will not be observed. The question then is whether the joint distribution of the unordered lifetimes of the m components $(U_1,U_2,\ldots,U_m)$ is identifiable or not from that of $X_{(1)},X_{(2)},\ldots,X_{(m-k+1)}$. If the component lifelength distributions are not identifiable, we propose bounds which can be estimated from the corresponding data. However, these bounds are more meaningful for a parallel system, i.e., the $k=1$ case.