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Optimal simple step stress accelerated life test design for reliability prediction

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ABSTRACT

A step stress accelerated life testing model is presented to obtain the optimal hold time at which the stress level is changed. The experimental test is designed to minimize the asymptotic variance of reliability estimate at time ζ. A Weibull distribution is assumed for the failure time at any constant stress level. The scale parameter of the Weibull failure time distribution at constant stress levels is assumed to be a log-linear function of the stress level. The maximum likelihood function is given for the step stress accelerated life testing model with Type I censoring, from which the asymptotic variance and the Fisher information matrix are obtained. An optimal test plan with the minimum asymptotic variance of reliability estimate at time ζ is determined.

1. Introduction

Reliability testing is commonly designed for life data analysis in which units are tested to failure in order to obtain failure time data for life data analysis. Accelerated life testing (ALT) is commonly practiced in product life testing and analysis. ALT is used to shorten the period between product design and release time and to improve the product performance and reliability. The determination of the warranty period for minimizing the warranty costs, increasing customer satisfaction, and maintaining scheduling are among the objectives of ALT. Under ALT, units are tested at higher-than-operating levels of stress (e.g., temperature, vibration, voltage, pressure, humidity, cycling rate, etc.) to induce early failures. Failure data collected from ALT are then extrapolated to obtain estimates for product characteristics, such as MTTF and reliability. Step-stress accelerated life testing (SSALT) is a special type of ALT in which stress levels are increased during the test period in a specified discrete sequence.

This paper presents an SSALT model for Weibull failure time data. The test is subject to termination at specified time T, leading to Type I censored failure data. The scale parameter of the Weibull distribution is assumed to be a log-linear function of the stress variable. The stress level remains fixed at S_1 until a hold time τ. Then it is increased to a higher level S_2; the test is continued until the censoring time T. The model is developed to obtain an optimal hold time τ^*, while the asymptotic variance (AV) of the reliability estimate at normal operating condition is minimized.

The test design and analysis of SSALT have been extensively studied in the recent years. In the SSALT planning problems, a commonly used optimization criterion is to minimize the AV of the maximum likelihood estimate (MLE) of the logarithm of mean life or some percentile of life at a specified stress level. Miller and Nelson (1983) and Bai et al. (1989) presented a simple SSALT plans assuming exponential life distribution. Khamis and Higgins (1996) considered quadratic stress–life relation and derived the optimum three-step SSALT for the exponentially distributed Type I censored data. Khamis (1997) proposed an optimal m-step SSALT design with k stress variables, assuming complete knowledge of the stress–life relation with multiple stress variables. Yeo and Tang (1999) and Tang (2003) derived a simple SSALT for an optimum hold time under low stress and an optimum low stress level by taking the target acceleration factor into consideration. These studies were based on the assumption that the

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failure time follows exponential distribution because of its simplicity. However, the assumption of exponential time to failure may not be applicable in many cases. Due to the flexibility of the Weibull distribution, it is more reasonable to assume that the failure time distribution of the test units is Weibull. Bai and Kim (1993) presented an optimal simple step-stress accelerated life test for the Weibull distribution under Type I censoring. A log-linear relationship and the cumulative exposure (CE) model were assumed, and nomographs were used to find the optimal plan. Khamis and Higgins (1998) proposed a time transformation of the exponential CE model for analyzing Weibull SSALT data, which made the computation of Weibull model less complicated. Alhadeed and Yang (2002) obtained the optimal design for the simple SSALT using the Khamis–Higgins (K–H) model, assuming constant shape parameter and no censoring was considered in their papers. Li and Fard (2007) developed a SSALT for two stress variables considering censored Weibull distributed failure data.

Reliability prediction in a product design and during the developmental testing process is an important factor. In order to accurately estimate the product reliability, the test design criterion is defined to minimize the AV of the reliability estimate at a time ς under normal operating condition. A few studies have focused on the reliability estimates as a test criterion. Elsayed and Zhang (2004) determined the optimum simple step-stress ALT plans based on Cox's proportional hazards model with a linear baseline hazard function. The optimization criterion was used to minimize the AV of the reliability function over a pre-specified period of time at a design stress level.

This paper proposes an optimal simple SSALT design considering reliability prediction for Type I censored Weibull failure data. It is assumed that the failure time of the test units follows Weibull distribution under constant stress level, and the K–H model is assumed to relate the cumulative density function (CDF) of failure time under constant-stress ALT to the CDF under step-stress ALT. Optimum design can be defined when predetermined values of the parameters are available. The optimization criterion is defined to minimize the AV of the reliability estimate at a specified time ς under normal operating condition. In the following section, the model and basic assumptions are presented first. Section 3 proposes the optimization criterion for optimal test design. In Section 4, the maximum likelihood function and the Fisher information matrix are given. The MLE method is used for the parameter estimation and analyses of the failure times t_{ij} from the SSALT, followed by an example in the last section.

2. Model assumptions

Initially n units are tested at a lower stress level S_1. The test is run until time τ, also known as hold time, when the stress level is increased to S_2. The test is continued until all units fail or until a predetermined censoring time T, whichever occurs first. The test procedure is shown in Fig. 1. Total of n_i failures are observed at time $t_{ij}, j = 1, 2, \ldots, n_i$, while testing at stress level S_i, $i = 1, 2$, and n_c units remain unfailed and censored at time T.

Assumptions. (i) Two stress levels S_1 and S_2 ($S_1 < S_2$) are used in the test, and S_0 is the stress level under normal operating conditions.

(ii) Under any constant stress, the time to failure of a test unit follows a Weibull distribution with distribution function:

$$G(t) = 1 - \exp\left(-\frac{t^\delta}{\theta_i^\delta}\right), \quad 0 \leqslant t < \infty, \quad i = 0, 1, 2, \tag{1}$$

where δ is the shape parameter and θ_i is the scale parameter.

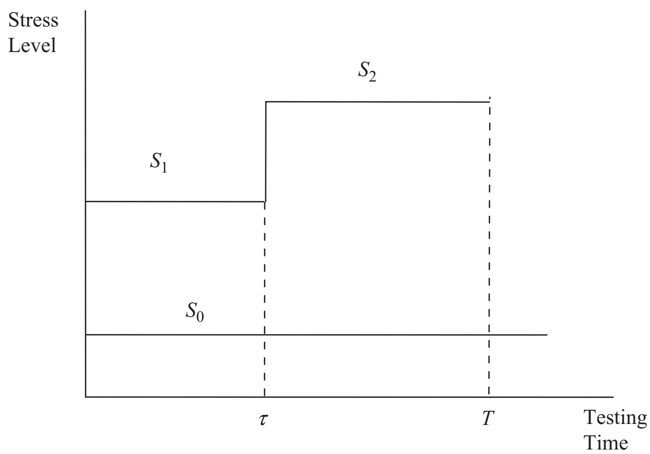

Fig. 1. Step-stress accelerated test model.