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Nonparametric multiple comparison procedures for unbalanced one-way factorial designs

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Abstract

In this paper, we present several nonparametric multiple comparison (MC) procedures for unbalanced one-way factorial designs. The nonparametric hypotheses are formulated by using normalized distribution functions and the comparisons are carried out on the basis of the relative treatment effects. The proposed test statistics take the form of linear pseudo rank statistics and the asymptotic joint distribution of the pseudo rank statistics for testing treatments versus control satisfies the multivariate totally positive of order two condition irrespective of the correlations among the rank statistics. Therefore, in the context of MCs of treatments versus control, the nonparametric Simes test is validated for the global testing of the intersection hypothesis. For simultaneous testing of individual hypotheses, the nonparametric Hochberg stepup procedure strongly controls the familywise type I error rate asymptotically. With regard to all pairwise comparisons, we generalize various single-step and stagewise procedures to perform comparisons on the relative treatment effects. To further compare with normal theory counterparts, the asymptotic relative efficiencies of the nonparametric MC procedures with respect to the parametric MC procedures are derived under a sequence of Pitman alternatives in a nonparametric location shift model for unbalanced one-way layouts. Monte Carlo simulations are conducted to demonstrate the validity and power of the proposed nonparametric MC procedures.

Keywords: Multiple comparisons; Multivariate totally positive of order two condition; Relative treatment effects; Simes inequality; Linear pseudo rank statistics; Stepdown procedures; Stepup procedures

1. Introduction

It has been well documented that the classical Bonferroni method is often very conservative with low power, especially for test statistics with highly dependent structure. Among the improved Bonferroni methods, Simes' (1986) test for the global testing of intersection hypothesis has received considerable interest. Hommel (1988) developed a multiple test procedure based on the closure principle. Hochberg (1988) proposed a stepwise procedure for simultaneous testing on individual hypotheses. The fact that these two procedures control the familywise error rate can be proved using the Simes inequality. Rom (1990) improved the power of Hochberg's stepup procedure by sharpening the sequentially

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rejective thresholds for the p-values. In order to establish the Simes conjecture for dependent statistics, Hochberg and Rom (1995), Chang et al. (1996), Samuel-Cahn (1996) have proved that the Simes test controls the overall type I error rate for positively associated bivariate distributions. Later, Sarkar and Chang (1997), and Sarkar (1998) have extended the validity of Simes' conjecture from the special case of a bivariate distribution to a general class of multivariate distributions satisfying the multivariate totally positive of order two (MTP2) condition.

Consider the problem of multiple comparisons (MCs) of treatments versus a control in an unbalanced one-way layout. As noted by Sarkar and Chang (1997), the parametric test statistics for comparing treatments versus one control with known error variance are conditionally independent given the parametric estimate of the effect of the control group. Therefore, the null distribution of such test statistics conforms to the MTP2 condition. When the error variance is unknown, the Simes conjecture holds as well for the resulting central multivariate t distribution (Sarkar, 1998). Note that the parametric test statistics use the critical points from the extreme tail portion of the t-distribution, which is the portion most sensitive to nonnormality. Therefore, when the normality assumption is violated, the problem of robustness will be more serious in the MC setting as compared to the setting of single inference (Hochberg and Tamhane, 1987). Thus it is desirable to develop nonparametric MC procedures with type I error rates which are independent of the underlying distributions. Furthermore, with respect to power, the nonparametric procedure is expected to possess power robustness compared to its parametric counterpart for a broad class of nonnormal distributions.

Nonparametric MC procedures have been developed for balanced factorial designs. Nemenyi (1963) proposed to apply Scheffé's (1953) method on the Friedman test and presented a conservative single-step procedure based on within-block rankings. Sen (1966, 1969) further developed single-step procedures analogous to Scheffé's method and Tukey's method (1953) based on aligned rank statistics and linear rank statistics, which can utilize inter-block rankings and thereby possess power advantage over Friedman-type statistics. Campbell and Skillings (1985) proposed stepdown procedures to test for the subset homogeneity hypotheses by employing separate ranking, joint ranking and the Kruskal–Wallis test statistic for unbalanced one-way layouts. However, the existing rank statistics are restricted to continuous distributions and thus cannot accommodate either ordinal data or ties which naturally arise in data analysis. Furthermore, it is unclear whether or not the existing test statistics satisfy the MTP2 condition so that the Simes test for global testing of intersection hypothesis and the Hochberg's procedure for simultaneous testing of individual hypotheses can be validated in the nonparametric context. There also exist few nonparametric stepwise procedures to perform all pairwise comparisons due to the complicated covariance structure of the test statistics. Furthermore Hochberg and Tamhane (1987, p. 249) and Hsu (1996, p. 177) pointed out that most of the traditional MC rank procedures only have a weak control of the familywise type I error rate due to the lack of the testing family property.

To address the aforementioned problem, we propose conducting nonparametric MCs under a broader nonparametric framework (Akritas and Arnold, 1994; Akritas et al., 1997). In this pure nonparametric setup, the hypotheses are formulated by means of distribution functions. The advantage of this nonparametric setup resides in the fact that the hypotheses in the linear model are implied by these nonparametric hypotheses. Furthermore the continuous distribution assumption is relaxed and ordinal data and ties are accommodated in this unified framework. Under these hypotheses, the asymptotic covariance structure of the rank means has a simple form and can be estimated by the ranks of the observations, which facilitates the development of multiple comparison procedures. To address the unbalance in these designs, we adopt the definition of the unweighted relative treatment effects, which has the advantage of not being influenced by the allocation of sample sizes in the data. The concept of an unweighted relative treatment effect was originally proposed by Brunner and Puri (1996), and Brunner and Munzel (2002), and the asymptotic properties of such rank statistics have been investigated in Domhof (2001). The estimates of the unweighted relative treatment effects take the form of the linear pseudo rank statistics, which has been employed in Gao and Alvo (2005) to test for main and interaction effects for unbalanced designs in non/semi-parametric location models.

In this article, we aim to develop a nonparametric analogue of Simes' test for the global testing of the intersection hypothesis involving MCs of treatments versus a control and also a nonparametric analogue of Hochberg's stepup procedure for the simultaneous testing of individual hypotheses. In contrast with the normal theory setting, where the parametric estimates for treatment effects are often independent, the nonparametric estimates of relative treatment effects are correlated. It is not readily obvious whether or not the null distribution of such rank statistics belongs to the family of distributions satisfying the MTP2 condition. Despite this inherent difficulty, through some asymptotic equivalence results, we are able to show that asymptotically the vector of pseudo rank statistics for individual comparisons are conditionally independent given a common variable associated with the control group. Consequently, the proposed rank statistics satisfy the MTP2 condition asymptotically. Furthermore, the proposed rank statistics are based on