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Robust likelihood functions in Bayesian inference

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Abstract

In order to deal with mild deviations from the assumed parametric model, we propose a procedure for accounting for model uncertainty in the Bayesian framework. In particular, in the derivation of posterior distributions, we discuss the use of robust pseudo-likelihoods, which offer the advantage of preventing the effects caused by model misspecifications, i.e. when the underlying distribution lies in a neighborhood of the assumed model. The influence functions of posterior summaries, such as the posterior mean, are investigated as well as the asymptotic properties of robust posterior distributions. Although the use of a pseudo-likelihood cannot be considered orthodox in the Bayesian perspective, it is shown that, also through some illustrative examples, how a robust pseudo-likelihood, with the same asymptotic properties of a genuine likelihood, can be useful in the inferential process in order to prevent the effects caused by model misspecifications.

Keywords: Estimating equation; Influence function; Kullback–Leibler divergence; Model misspecification; Pseudo-likelihood; Posterior distribution; Robustness

1. Introduction

In the frequentist framework, it is well known that mild deviations from the assumed model can give rise to non-negligible changes in inferential results. The concept of robustness related to likelihood functions has been discussed under various points of views in the literature; see, for instance, Heritier and Ronchetti (1994), Tsou and Royall (1995), Lavine (1995) and Markatou et al. (1998). This issue is also addressed in the approach based on the influence function (IF) discussed in Huber (1981) and Hampel et al. (1986), in which unbiased estimating equations are claimed to supply effective tools for robust inference. In this context, robust pseudo-likelihoods can be derived from bounded estimating functions and can be used as genuine likelihoods. In particular, in this paper we focus our attention on the quasi- and the empirical likelihoods (see e.g. Owen, 2001; Adimari and Ventura, 2002a, b), which on the one hand keep the standard first-order properties, and on the other hand take into account model inadequacies. Indeed, they provide inferential procedures which are still reliable and reasonably efficient when the underlying distribution lies in a neighborhood of the assumed model.

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Obviously, even in the Bayesian approach, inferential results depend on the modeling assumptions and on the observed sample, but Bayesian robust statistic is mainly concerned with the global and local sensitivity to prior distributions; see, for instance, Berger (1994), Rios Insua and Ruggeri (2000) and references therein. Indeed, the problem of robustness with respect to sampling model misspecifications has been considered only in few contributions (Lavine, 1991; Sivaganesan, 1993; Dey et al., 1996; Gustafson, 1996; Shyamalkumar, 2000). Other exceptions are discussed in Pericchi and Perez (1994), Pericchi and Sansò (1995), David (1973), O'Hagan (1979, 1988, 1990), Verdinelli and Wasserman (1991), Passarin (2004) and Andrade and O'Hagan (2006), who address the topics of outliers, robust posterior summaries and model embedding.

The solution of embedding in a larger structure has the cost of eliciting a prior distribution for the extra parameters introduced in the analysis. Moreover, the statistical procedures derived under the supermodel are not necessarily robust in a broad sense, the supermodel being too thin in the space of all distributions. In this paper, we focus on an alternative approach and we explore the use of the quasi- and the empirical likelihoods mentioned above. Actually, since these robust pseudo-likelihoods share most of the properties of the genuine likelihood, they can be used as a basis for Bayesian inference to obtain a "robust posterior distribution". Although this way of proceeding on principle cannot be considered as orthodox in the Bayesian perspective, it is of interest to evaluate whether the use of a robust likelihood may be useful in the inferential process. Related works on the use of alternative likelihoods in Bayesian inference are in Efron (1993), Bertolino and Racugno (1992, 1994), Raftery et al. (1996) and Cabras et al. (2006). Papers which are more specifically related to the validation of a pseudo-posterior distribution based on an alternative likelihood function are Monahan and Boos (1992), Severini (1999), Lazar (2003), Racugno et al. (2005), Pace et al. (2006) and Schennach (2005).

We discuss two properties of robust posterior distributions, which are derived from the corresponding properties of the robust pseudo-likelihoods involved. First of all, we show that the robust posterior distributions are asymptotically normal as the genuine posterior distributions and that the asymptotic Kullback–Leibler divergence between a robust posterior distribution and the corresponding genuine posterior distribution is related to the required level of robustness. Moreover, to assess the stability of robust posterior inference, we show that the IF of summaries of the robust posterior distributions are bounded, contrarily of those based on the genuine posterior distribution. A bounded IF is a desirable local stability property of an inferential procedure since it implies that, for each sample size, in a neighborhood of the assumed model the effect of a small contamination on the procedure cannot become arbitrarily large. The need for bounded influence statistical procedures for estimation, testing and prediction has been stressed by many authors (see, for instance, Hampel et al., 1986; Peracchi, 1990, 1991; Markatou and Ronchetti, 1997) and the aim of this paper is to contribute to the current literature in the Bayesian direction.

A motivating example: Let us consider the well-known Darwin's paired data (Spiegelhalter, 1985) on the heights of self- and cross-fertilized plants. This example is discussed, in the Bayesian literature, also by Pericchi and Perez (1994) and Shyamalkumar (2000). It is pointed out that there is not enough evidence to distinguish between the normal model and a distribution with heavy tails, such as the Cauchy and the Student's t distributions, the latter being slightly preferred. Fig. 1(a) shows the normal probability plot of these data. Interest focuses on the location parameter of the distribution of the difference of the heights.

By modeling likelihood uncertainty by the normal, Cauchy and Student's t_3 distributions and assuming the same default noninformative prior distribution for the location parameter, we obtain the posterior distributions plotted in Fig. 1(b). Notice that posterior inferential results can depend strongly on the assumed model.

Even though simple, this example is interesting in its results since it illustrates in a practical situation the problem we deal with. In order to handle with model uncertainty, we account for the imprecision with respect to the likelihood by considering robust pseudo-likelihoods in the Bayesian practice. Their main appeal is that they allow to assume the simpler sampling model and to deal anyhow with mild departures from it. The proposed approach leads to directly embed model uncertainty in the likelihood function and, in view of this, to arrive at posterior inferential conclusions which prevent the effects caused by model misspecifications, i.e. when the underlying distribution lies in a neighborhood of the simpler assumed model.

The paper is organized as follows. Section 2 presents a short overview of M-estimators, IF and quasi- and empirical likelihood functions. Section 3 discusses robust posterior distributions, focusing on their asymptotic properties and on the computation of the IF of posterior summaries. Section 4 illustrates by explanatory examples and Monte Carlo studies the use of robust pseudo-likelihoods in the Bayesian framework. Finally, some conclusive remarks are given in Section 5.