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Semiparametric estimation of spatial long-range dependence

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Abstract

Estimation of the long-range dependence parameter in spatial processes using a semiparametric approach is studied. An extended formulation of the averaged periodogram method proposed in Robinson [1994. Semiparametric analysis of long memory time series. Ann. Statist. 22, 515–539] is derived, considering a certain homogeneous and isotropic behaviour of the spectral distribution in the low frequencies. The weak consistency of the estimator proposed is proved.

Keywords: Averaged periodogram; Semiparametric estimation; Spectral distribution; Strong spatial dependence; Weak consistency

1. Introduction

Temporal and spatial data displaying long-range dependence are often involved in the statistical analysis of phenomena of interest in many applied fields. For example, long-range correlation models have been considered for return processes in stock market (see Lo, 1991; Lobato and Velasco, 2000, among others). Spatial strong-dependence models have also been formulated in the analysis of spatial variability of soil properties, air ozone concentration, velocity turbulence fields, etc. (see, for instance, Akkaya and Yücemen, 2002; Anh et al., 2000; Marguerit et al., 1998).

Different approaches have been introduced to derive suitable models representing long memory and strong dependence in temporal and spatial data. Specifically, in the temporal case, fractional time series models (see Beran, 1994, Chapter 2) and regression models (see Geweke and Porter-Hudak, 1983) are considered as a flexible framework to represent long-correlation data. Extended long-correlation models on a lattice have been defined, for example, in Heyde and Yang (1997) and Ma (2003). The theory of self-similar and fractal fields also provides a suitable context for the introduction of strong-dependence spatial process models (see Anh et al., 1999; Kelbert et al., 2005; Leonenko and Anh, 2001).

Estimation methods for long-range dependence parameters have been widely studied within the theory of fractional time series. Semiparametric procedures based on the periodogram have been formulated, for example, in Beran (1994) and Robinson (1994, 1995a, b). Wavelet-based estimates have been considered, for instance, in Abry et al. (1998), Bardet et al. (2003), Pesquet-Popescu and Lévy-Véhel (2002), and Wornell and Oppenheim (1992). Variogram-based estimates in self-similar models are also investigated in Chan and Wood (2000), among others. The continuous-time formulation

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of the above-mentioned estimation approaches follows by slight modifications. However, their extensions to the spatial or spatio-temporal statistical context involve a higher complexity depending on the geometrical characteristics of the spatial covariance model considered (see, for example, Frias et al., 2006). For instance, the adaptation to the separable spatial covariance case can be straightforwardly derived. For general covariance models, some difficulties arise since the extension involves spherical formulation and spatial geometrical adaptation of estimation procedures.

In this paper, we extend the semiparametric estimation method proposed by Robinson (1994) for long-memory time series to the spatial case. In Section 2, we introduce the class of spatial long-range dependence processes studied. The conditions assumed as well as the long-memory parameter estimator are also formulated. The theoretical results needed for the convergence in probability of the proposed estimator are proved in Section 3. Conclusions and discussion on further extensions of the results are given in Section 4. Auxiliary results are collected in the Appendix.

2. The estimation method

Let $\{X(\mathbf{s}) : \mathbf{s} \in \mathbb{R}^d\}$ be a homogeneous random field with covariance function C and associated spectral distribution G, defining a non-negative finite measure on $\mathbb{R}^d$. In the case where G is absolutely continuous on $\mathbb{R}^d$, we denote by

$$f(\lambda_1, \lambda_2, \ldots, \lambda_d) = \frac{\partial^d G(\lambda_1, \lambda_2, \ldots, \lambda_d)}{\partial\lambda_1 \partial\lambda_2 \ldots \partial\lambda_d}$$

the spectral density function. We also consider the function

$$F(\lambda_1, \lambda_2, \ldots, \lambda_d) = \int_0^{\lambda_1} \int_0^{\lambda_2} \cdots \int_0^{\lambda_d} f(\xi_1, \xi_2, \ldots, \xi_d)\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \ldots \mathrm{d}\xi_d$$

defined from the restriction of f to positive frequencies.

In the definition below, the concept of 'slowly varying function' at infinity is used, referring to a function $L : (0, \infty) \to (0, \infty)$ satisfying, for all $c > 0$,

$$\lim_{t \to \infty} \frac{L(ct)}{L(t)} = 1.$$

Definition 1. Let $\{X(\mathbf{s}) : \mathbf{s} \in \mathbb{R}^d\}$ be a homogeneous and isotropic random field with covariance function $C(\mathbf{s}_1, \mathbf{s}_2) = \mathrm{cov}(X(\mathbf{s}_1), X(\mathbf{s}_2)) = C(|\mathbf{s}_1 - \mathbf{s}_2|) = C(|\mathbf{h}|)$ satisfying

$$\lim_{|\mathbf{h}| \to \infty} \frac{C(|\mathbf{h}|)}{L(|\mathbf{h}|)|\mathbf{h}|^{-d-1+2H}} = 1, \quad \frac{1}{2} < H < \frac{d+1}{2}, \tag{1}$$

where L is a slowly varying function at infinity. Then, random field X is said to display long-range dependence.

Under Tauberian-type theorem conditions, random fields satisfying (1) also have the frequency domain property

$$\lim_{|\boldsymbol{\lambda}| \to 0} \frac{f(|\boldsymbol{\lambda}|)}{L(1/|\boldsymbol{\lambda}|)|\boldsymbol{\lambda}|^{1-2H}} = 1, \quad \frac{1}{2} < H < \frac{d+1}{2}. \tag{2}$$

Then $f(|\boldsymbol{\lambda}|)$ has an integrable pole at the origin. Under Abelian-type theorem conditions, statement (2) implies (1) (see, for example, Leonenko, 1999).

In the following, we refer the study to the case $d = 2$.

In the second-order statistical analysis of spatial data, namely, in the implementation of estimation procedures based on the spectral domain, the periodogram plays a fundamental role. Here, we consider its biparametric formulation, $I(\boldsymbol{\lambda})$, given by

$$I(\boldsymbol{\lambda}) = |\omega(\boldsymbol{\lambda})|^2,$$

$$\omega(\boldsymbol{\lambda}) = (4\pi^2 n)^{-1/2} \sum_{s_1=1}^{n_1} \sum_{s_2=2}^{n_2} X(s_1, s_2) \mathrm{e}^{\mathrm{i}(s_1\lambda_1 + s_2\lambda_2)},$$