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ScienceDirect
Journal of Statistical Planning and Inference 137 (2007) 1043–1058
journal of statistical planning and inference
www.elsevier.com/locate/jspi

Higher order asymptotic option valuation for non-Gaussian dependent returns

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Available online 25 July 2006

Abstract

This paper discusses the option pricing problems using statistical series expansion for the price process of an underlying asset. We derive the Edgeworth expansion for the stock log return via extracting dynamics structure of time series. Using this result, we investigate influences of the non-Gaussianity and the dependency of log return processes for option pricing. Numerical studies show some interesting features of them.

MSC: Primary 91B24; 62M10; secondary 62M15; 91B84

Keywords: Black and Scholes model; Edgeworth expansion; Non-Gaussian stationary process; Option pricing

1. Introduction

Black and Scholes (1973) provided the foundation of modern option pricing theory. Despite its usefulness, however, the Black and Scholes theory entails some inconsistencies. It is well known that the model frequently misprices deep in-the-money and deep out-of-the-money options. This result is generally attributed to the unrealistic assumptions used to derive the model. In particular, the Black and Scholes model assumes that stock prices follow a geometric Brownian motion with a constant volatility under an equivalent martingale measure.

In order to avoid this drawback, Jarrow and Rudd (1982) proposed a semiparametric option pricing model to account for non-normal skewness and kurtosis in stock returns. This approach aims to approximate the risk-neutral density by a statistical series expansion. Jarrow and Rudd (1982) approximated the density of the state price by an Edgeworth series expansion involving the log-normal density. Corrado and Su (1996a) implemented Jarrow and Rudd's formula to price options. Corrado and Su (1996b, 1997) considered Gram-Charlier expansions for the stock log return rather than the stock price itself. Rubinstein (1998) used the Edgeworth expansion for the stock log return. Jurczenko et al. (2002) compared these different multi-moment approximate option pricing models. Also they investigated in particular the conditions that ensure the martingale restriction.

As in Kariya (1993) and Kariya and Liu (2002), the time series structure of return series does not always admit a measure which makes the discounted process a martingale. Hence, we will not be able to develop an arbitrage pricing theory by forming an equivalent portfolio. In such a case, we often regard the expected value of the present value

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0378-3758/$ - see front matter
doi:10.1016/j.jspi.2006.06.023

of a contingent claim as a proxy for pricing may be with help of a risk neutrality argument. In view of this, Kariya (1993) considered pricing problems with no martingale property and approximated the density of the state price by the Gram-Charlier expansion for the stock log return.

In this paper, we consider option pricing problems by using Kariya's approach. In Section 2, we derive the Edgeworth expansion for the stock log return via extracting dynamics structure of time series. Using this result, we investigate influences of the non-Gaussianity and the dependency of log return processes for option pricing. Numerical studies illuminate some interesting features of the influences. In Section 3, we give option prices based on the risk neutrality argument. In Section 4, we discuss a consistent estimator of the quantities in our results. Section 5 concludes. The proofs of theorems are relegated to Section 6.

2. Edgeworth expansion of log return

Let $\{S_t; t \geqslant 0\}$ be the price process of an underlying security at trading time t. The j-th period log return X_j is defined as

$$\log S_{T_0+j\varDelta} - \log S_{T_0+(j-1)\varDelta} = \Delta\mu + \varDelta^{1/2} X_j, \quad j = 1, 2, \ldots, N, \tag{1}$$

where T_0 is present time, $N = \tau/\varDelta$ is the number of unit time intervals of length $\varDelta$ during a period $\tau = T - T_0$ and T is the maturity date. Then the terminal price S_T of the underlying security is given by

$$S_T = S_{T_0} \exp\left\{\tau\mu + \left(\frac{\tau}{N}\right)^{1/2} \sum_{j=1}^{N} X_j\right\}. \tag{2}$$

Remark 1. In the Black and Scholes option theory the price process is assumed to be a geometric Brownian motion

$$S_T = S_{T_0} \exp(\tau\mu + \sigma W_\tau), \tag{3}$$

where the process $\{W_t; t \in \mathbf{R}\}$ is a Wiener process with drift 0 and variance t. From (3), the log return at discreterized time point can be written as

$$\log S_{t+j\varDelta} - \log S_{t+(j-1)\varDelta} = \Delta\mu + \varDelta^{1/2}\sigma v_j, \quad v_j \sim \text{iid N}(0, 1). \tag{4}$$

The expression of (1) is motivated from (4).

First, we derive an analytical expression for the density function of S_T. Since from (2) the distribution of S_T depends on that of $Z_N = N^{-1/2}\sum_{j=1}^{N} X_j$, we consider the Edgeworth expansion of the density function of Z_N. If we assume that X_j are independently and identically distributed random variables with mean zero and finite variance, it is easy to give the Edgeworth expansion for Z_N (the classical Edgeworth expansion).

However, a lot of financial empirical studies show that X_j's are not independent. Thus, we suppose that $\{X_j\}$ is a dependent process which satisfies the following assumption.

(A1) $\{X_t; t \in \mathbf{Z}\}$ is fourth order stationary in the sense that

(i) $E(X_t) = 0$,
(ii) $\mathrm{cum}(X_t, X_{t+u}) = c_{X,2}(u)$,
(iii) $\mathrm{cum}(X_t, X_{t+u_1}, X_{t+u_2}) = c_{X,3}(u_1, u_2)$,
(iv) $\mathrm{cum}(X_t, X_{t+u_1}, X_{t+u_2}, X_{t+u_3}) = c_{X,4}(u_1, u_2, u_3)$.

(A2) The cumulants $c_{X,k}(u_1, \ldots, u_{k-1})$, $k = 2, 3, 4$, satisfy

$$\sum_{u_1,\ldots,u_{k-1}=-\infty}^{\infty} (1 + |u_j|^{2-k/2})|c_{X,k}(u_1, \ldots, u_{k-1})| < \infty$$

for $j = 1, \ldots, k-1$.

(A3) J-th order ($J \geqslant 5$) cumulants of Z_N are all $\mathrm{O}(N^{-J/2+1})$.