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Connection between uniformity and orthogonality for symmetrical factorial designs

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Abstract

In this paper, we study the issue of uniformity in symmetrical fractional factorial designs. The discrete discrepancy (Biometrika 89 (2002) 893; Metrika 58 (2003) 279; Metrika 60 (2004) 59) is employed as a measure of uniformity. Although there are some emerging literature for connecting uniformity with orthogonality, less attention has been given to this issue for more than three-level fractional factorials and asymmetric fractional factorials. This paper discusses this issue for general symmetric fractional factorials. We derive results connecting uniformity and orthogonality and show that these criteria agree quite well, which provide further justifiable interpretation for some criteria of orthogonality by the consideration of uniformity. In addition, we also point that two measures of orthogonality in the literature (Fang, Hickernell, Niederreiter (Eds.), Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing, Springer, Berlin, 2002; J. Complexity 19 (2003) 692) are equivalent and derive now a lower bound of the discrete discrepancy.

MSC: 62K15; 62K10; 62K99

Keywords: Discrete discrepancy; Orthogonality; Uniformity; Uniform design

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1. Introduction

Uniform design (Wang and Fang, 1981; Fang and Wang, 1994; Fang et al., 2000) spreads its experimental points evenly throughout the design space so that one can explore a variety of models. Uniformity is an important concept in uniform designs. Recently, considerable effort has been taken for exploring applications of the uniformity in factorial designs. Fang et al. (2000) gave a connection between orthogonal design and uniform designs under the centered L_2-discrepancy (Hickernell, 1998), and conjectured that any orthogonal designs is a uniform design under a certain measure of uniformity. Ma et al. (2003) proved this conjecture to be true for a complete design q^s if $q = 2$ or q is odd, or $s = 1$ or 2. For two- or three-level fractions factorials, Fang et al. (2002) derived results connecting orthogonality and uniformity measured by some currently popular criteria. Chatterjee et al. (2005) extended their results to asymmetric factorials with two and three levels. This connection between uniformity and orthogonality provides an additional rationale for using uniform designs.

Recently, one new measure of uniformity was proposed by Hickernell and Liu (2002) and discussed by Liu and Hickernell (2002), Liu (2002) and Qin and Fang (2004), that is, the *discrete discrepancy* (DD, for short). One main advantage of the DD is to investigate the relationship between runs rather than the factors. Compared to others, the DD not only enormously reduces the computational cost but also has statistical properties. Hickernell and Liu (2002) showed that uniform designs under the DD limit the effects of aliasing to yield reasonable efficiency and robustness together. Liu and Hickernell (2002) employed the DD to assess supersaturated experimental designs and showed that for two-level supersaturated designs the DD and the popular $E(s^2)$ criterion share the same optimal designs. Liu (2002) used the projection of DD to evaluate the uniformity of symmetrical fractional factorials and found that orthogonality and uniformity are strongly related to each other. Qin and Fang (2004) reported that the DD can be regarded as a kind of generalization of other currently popular discrepancies, and gave linkages among uniformity measured by the DD and other criteria for optimal factor assignment, whose close linkages provide a significant justification for the DD used to measure uniformity of factorial designs. In addition, Qin and Fang (2004) implicitly proved symmetrical saturated orthogonal designs and a special class of symmetrical supersaturated designs being uniform designs under the DD.

While the work of Fang et al. (2002) was a first attempt towards connecting uniformity and orthogonality through an analytic linkage for two- or three-level factorials, the present paper aims at obtaining further results. We extend the findings in Fang et al. (2002) to general symmetrical factorials. First, the DD is utilized as a measure of uniformity, which can be regarded as a kind of generalization of measures of uniformity discussed by Fang et al. (2002), see Theorem 5 of Qin and Fang (2004). Second, some results reported in this paper can reduce to that of Fang et al. (2002) for two- or three-level factorials. Third, unlike Liu and Hickernell (2002) and Liu (2002), we build some analytic linkages between uniformity and orthogonality. Finally, the uniformity under the DD cannot be expressed by the form given by Fang et al. (2002), thus the techniques of proof as in Fang et al. (2002) do not work here and new techniques are employed in this paper.

The rest of this paper is organized as follows. In Section 2, the discrete discrepancy, orthogonal array and two criteria which can be viewed as extensions of strength of orthogonal