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Journal of Statistical Planning and Inference 136 (2006) 1493–1511
journal of statistical planning and inference
www.elsevier.com/locate/jspi

Minimum aberration blocking of regular mixed factorial designs

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Received 13 January 2003; accepted 1 October 2004
Available online 5 November 2004

Abstract

It is known by Zhang and Park (J. Statist. Plann. Inference 91 (2000) 107) that there are no minimum aberration (MA) designs with respect to both treatments and blocks for blocked regular mixed-level factorial designs. So it should be compromised between the block wordlength pattern and treatment wordlength pattern. Two methods are considered in this article. The first is MA blocking scheme of an MA design. The other is to combine the components of the two wordlength pattern vectors into one combined wordlength pattern according to the modified hierarchical assumptions and an appropriate ordering of the numbers of alias or confounding relations. The relationship between the two types of optimal blocked designs is investigated. A complete catalogue of optimal blocked regular mixed factorial designs of the above two types with 16 or 32 runs is given.

MSC: 62K15; 62K10; 62K05

Keywords: Blocking; Fractional factorial design; Minimum aberration; Mixed-level; Regular

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doi:10.1016/j.jspi.2004.10.001

1. Introduction

A factorial design in which the numbers of levels of the factors are not all equal is called an asymmetrical or mixed-level factorial design. The regular mixed factorial designs can be easily constructed from the corresponding symmetrical designs through the method of replacement, which was first introduced by Addelman (1962). Wu (1989) improved Addelman's construction method by introducing the method of grouping. Wu et al. (1992) further extended the method of grouping to more general designs and constructed a large class of asymmetrical orthogonal arrays. Subsequently, Wu and Zhang (1993) extended minimum aberration (MA) criterion (Fries and Hunter, 1980) to select optimal mixed-level factorial designs.

Arranging a mixed-level factorial design into s^k blocks is equivalent to selecting k independent columns $u_1, \ldots, u_k$ for the k block factors $b_1, \ldots, b_k$, i.e. $b_1 = u_1, \ldots, b_k = u_k$, which is called a blocking scheme. The group formed by the k block factors, denoted by $G_b = \{\mathbf{I}, u_1, \ldots, u_{L_k}\}$, is called the block defining contrast subgroup, where $L_u = (s^u - 1)/(s-1)$ for any nonnegative integer u and the subscript 'b' stands for 'block'.

As usual, we still take the following five assumptions:

(i) Lower-order interactions are more likely to be important than higher-order interactions.
(ii) Interactions of the same order and of the same type are equally likely to be important.
(iii) The interaction between two or more block factors has the same importance as a block main effect.
(iv) Block factors have no interaction with treatment factors.
(v) Block effects are more likely to be significant than treatment effects.

For a blocked fractional factorial design, let $\{w_1, \ldots, w_p, u_1, \ldots, u_k\}$ denote the p treatment generators and k block generators which are all independent. Let G_t and W_t be the defining contrast subgroup formed by the p treatment generators and the corresponding wordlength pattern, respectively. The group formed by the $p+k$ independent generators is denoted by G_{t+b}, where the subscript '$t+b$' stands for 'the overall consisting of both treatment and block'. Correspondingly, we can define its wordlength pattern and denote it by W_{t+b}. The set $G_{t+b} \backslash G_t$ is denoted by $G_{b\otimes t}$ which is the set of treatment effects confounded with block effects, where the '$b \otimes t$' stands for 'the confounding of block with treatment'. The wordlength pattern in $G_{b\otimes t}$, denoted by $W_{b\otimes t}$, indicates the influence of block factors on the treatment effects and is called the block wordlength pattern. Obviously, we have

$$G_{t+b} = G_t + G_{b\otimes t} \quad \text{and} \quad W_{t+b} = W_t + W_{b\otimes t}. \tag{1}$$

Thus, there are two wordlength patterns for blocked fractional factorial designs. Ideally, one should search for a design that has MA with respect to both treatments and blocks. However, Zhang and Park (2000) showed that there is no minimum aberration design with respect to both treatments and blocks. For this reason, it is needed to compromise between the treatment wordlength pattern and the block wordlength pattern.

Two methods may be considered. The first is to treat separately the two wordlength pattern vectors W_t and $W_{b\otimes t}$. Along this line, Sun et al. (1997) introduced a notion of admissibility. Since there are too many admissible designs, this approach does not completely resolve the