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Sequential testing of hypotheses about drift for Gaussian diffusions

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ABSTRACT

In statistical inference on the drift parameter θ in the process $X_t = \theta a(t) + \int_0^t b(s)\mathrm{d}W_s$, where $a(t)$ and $b(t)$ are known, deterministic functions, there is known a large number of options how to do it. We may, for example, base this inference on the differences between the observed values of the process at discrete times and their normality. Although such methods are very simple, it turns out that it is more appropriate to use sequential methods. For the hypotheses testing about the drift parameter θ, it is more proper to standardize the observed process and to use sequential methods based on the first exit time of the observed process of a pre-specified interval until some given time. These methods can be generalized to the case of random part being a symmetric Itô integral or continuous symmetric martingale.

1. Introduction

When we observe a stochastic process with a symmetric random noise and deterministic part, we often need to know the sign of the deterministic part. If, in addition, the magnitude of the trend is too large, we prefer to detect it as soon as possible. This sign is usually characterized by the sign (or magnitude) of the drift parameter. For statistical inference on the drift parameter, it is very important to find a proper way how to obtain a suitable statistical method. Since we are usually unable to observe the whole trajectory of such a process, we can observe the process only in a set of discrete time points or observe the time points when the process exits some open interval.

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In this paper, we work with the Itô integral of a known, deterministic function $b(t)$ with a deterministic drift $\theta\dot{a}(t)$, where $\dot{a}(t)$ stands for the derivative of a known function $a(t)$, i.e.

$$X_t = \theta a(t) + \int_0^t b(s)\mathrm{d}W_s. \tag{1}$$

The paper deals with estimating and hypotheses testing possibilities for the parameter θ.

An application of the process (1) can be quite large and it can be used for modelling purposes. For $b(t) \equiv 1$ and $a(t) = t$, the process (1) can be applied in finance, especially for option pricing (see [7,34,35,12], or [17]). Such a process can be also applied in modelling interest rates (see [43,10], or [1]) or in medicine, especially for modelling clinical trials (see [13,44], or [19]).

When observing the process (1) at discrete time points, the methods based on the generalized least squares method and maximum likelihood procedure have been proposed for more general or similar processes (see [21,29,38,4,20,26], or [45]).

When we are able to observe the whole trajectory of the process (1), we are also able to estimate the drift parameter by observing the whole process (1) until some given time. Such an approach has the advantage that it can be applied to more general processes. Then the parameter θ can be estimated by means of the maximum likelihood procedure based on the Girsanov theorem (see [24, Chapter 17]).

The main result of this paper is an application of sequential methods for hypotheses testing about the parameter θ in the process (1). In the past, the sequential methods have been considered in [9], who have applied the first hitting time of the observed process to a pre-specified boundary for estimating the drift parameter for the Wiener process with a constant drift. They have showed that the stopping time is a complete sufficient statistic for the drift parameter and with known distribution that belongs to an exponential family. They have also applied the stopping time for deriving the minimum variance unbiased estimator for the drift parameter. The sequential methods have been also considered in [27] and [24, 17.5], who have considered the sequential maximum likelihood estimator for estimating the drift parameter for diffusion processes. The sequential maximum likelihood estimator has been also extended in [6, 5.2] to infinite dimensional diffusions solving stochastic partial differential equations. For other similar processes, the sequential maximum likelihood estimators have been considered in [23] for reflected Ornstein–Uhlenbeck processes, in [8] for a class of reflected generalized Ornstein–Uhlenbeck processes driven by spectrally positive Lévy processes, or in [22] for the hyperbolic diffusion processes. For hypotheses testing about the drift parameter, the sequential probability ratio test has been considered in [2] and [36, 4.2] for testing the hypothesis that the observed process is just the Wiener process against the alternative that the process contains some specific nonzero drift. The sequential probability ratio test has been also considered in [3, 12.3] and [24, 17.6] for more general alternatives. The sequential probability ratio test has been also generalized to the case of delayed observations in [25] or [15]. As the sequential probability ratio test has been initially based on observation of whether the considered process (or some of its monotonous transformation) reaches some pre-specified upper boundary region before some pre-specified lower boundary region and without any bounded time of observation (without finite horizon), which can increase the average times of decision for some alternatives, the sequential methods have been also considered with finite horizon in [14], who has generalized the test by means of the parabolic boundary regions. Other sequential methods with finite horizon have been considered in [37, Chapter 3], [16], or [30, VI.21]. The advantage of the sequential probability ratio test or some of tests based on parabolic boundary regions is that the tests can be constructed in a way that they are optimal at given hypothesis and at given alternative, i.e. the average times of decision have the least values among all proper sequential tests with prescribed probabilities of type I and type II errors at given hypothesis and at given alternative. However, the average times of decision cannot have the least values uniformly for all alternatives, furthermore, they are greater for the sequential probability ratio test for smaller values of alternatives (see [14] for graphical comparison). Moreover, the sequential methods have been mostly considered just for the Wiener process with a constant drift and for hypotheses testing with simple or one-sided alternatives. Because of these restrictions, we have considered methods that do not need to be optimal at given values, but that can be applied to more general processes and to more general hypotheses testing, especially to hypotheses testing with two-sided alternatives.