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Exact likelihood inference for an exponential parameter under generalized progressive hybrid censoring scheme

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ABSTRACT

Recently, progressive hybrid censoring schemes have become quite popular in a life-testing problem and reliability analysis. However, the limitation of the progressive hybrid censoring scheme is that it cannot be applied when few failures occur before time T. In this article, we propose a generalized progressive hybrid censoring scheme, which allows us to observe a pre-specified number of failures. So, the certain number of failures and their survival times are provided all the time. We also derive the exact distribution of the maximum likelihood estimator (MLE) as well as exact confidence interval (CI) for the parameter of the exponential distribution under the generalized progressive hybrid censoring scheme. The results of simulation studies and real-life data analysis are included to illustrate the proposed method.

1. Introduction

In many life testing and reliability studies, it is well known that the lifetimes of test units may not be recorded exactly. There are also situations wherein the removal of units prior to failure is pre-planned in order to reduce the cost or time associated with testing. The most common censoring schemes are Type I and Type II censoring. The mixture of Type I and Type II censoring schemes is known as the hybrid censoring scheme. The hybrid censoring scheme was first introduced by Epstein [8]. Some recent studies on hybrid censoring have been carried out by many authors including Kundu [11], Dube et al. [6] and Balakrishnan and Kundu [4].

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If an experimenter desires to remove live experimental units at points other than the final termination point of the experiment, the Type I and Type II censoring schemes will not be of use. The conventional censoring schemes do not allow for units to be removed from the test at points other than the final termination point. Intermediate removal may be desirable when a compromise between reduced time of experimentation and the observation of at least some extreme lifetimes is sought, or when some of the surviving units in the experiment that are removed early on can be used for some other tests. Therefore, the loss of units at points other than the final termination point may be unavoidable, as in the case of accidental breakage of experimental units or loss of contact with individuals under experiment. These reasons and motivations lead reliability practitioners and theoreticians directly into the area of progressive censoring (Balakrishnan and Aggarwala, [2]). Some recent studies on progressive Type II censoring have been carried out by many authors including Rastogi and Tripathi [16], Ahmed [1], Huang and Wu [10], and Wu [18].

Progressive censoring scheme can be described as follows. Immediately following the first observed failure, R_1 surviving units are removed from the test at random. Similarly, following the second observed failure, R_2 surviving units are removed from the test at random. This process continues until, immediately following the mth observed failure, all the remaining $R_m = n - R_1 - \cdots - R_{m-1} - m$ units are removed from the experiment. In this experiment, the progressive censoring scheme $R = (R_1, R_2, \ldots, R_m)$ is pre-fixed. The resulting m ordered failure times, which we denote by $X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$, are referred to as progressive Type II censoring scheme. The joint probability density function of $(X_{1:m:n}, X_{2:m:n}, \ldots, X_{i:m:n})$ $(i = 1, 2, \ldots, m)$ is given by (Balakrishnan and Aggarwala, [2])

$$f(x_1, x_2, \ldots, x_i) = \left[\prod_{j=1}^{i} \sum_{k=j}^{m} (R_k + 1)\right] \prod_{j=1}^{i-1} f(x_j)[1 - F(x_j)]^{R_j} f(x_i)[1 - F(x_i)]^{R_i^* - 1}, \quad (1)$$

where $R_i^* = \sum_{k=i}^{m} (R_k + 1)$, $-\infty < x_1 < x_2 < \cdots < x_i < \infty$.

The disadvantages of the progressive Type II censoring scheme are that the time of the experiment can be very long if the units are highly reliable. Because of that, Kundu and Joarder [12] and Childs et al. [5] proposed a progressive hybrid censoring scheme in the context of life-testing experiment in which n identical units are placed on experiment with progressive Type II censoring scheme $(R_1, R_2, \ldots, R_m)$, and the experiment is terminated at time $\min\{X_{m:m:n}, T\}$, where $T \in (0, \infty)$ and $1 \le m \le n$ are fixed in advance, and $X_{1:m:n} \le X_{2:m:n} \le \cdots \le X_{m:m:n}$ are the ordered failure times resulting from the experiment. Under progressive hybrid censoring scheme the time on experiment will be no more than T. Some recent studies on progressive hybrid censoring have been carried out by many authors including Lin et al. [13], Lin and Huang [14], Lin et al. [15], and Hemmati and Khorram [9].

One limitation of the progressive hybrid censoring scheme is that it cannot be applied when very few failures may occur before time T. Therefore MLE for a parameter of an underling distribution of observations may not be computed or its accuracy will be extremely low. In this reason, we propose a generalized progressive hybrid censoring scheme, which allows us to observe a pre-specified number of failures. So, the certain number of failures and their lifetimes are always provided under the generalized progressive hybrid censoring scheme. The life-testing experiment based on the proposed censoring scheme can save both the total time on tests and the cost induced by failures of the units. Moreover, the efficiency of statistical estimation is increased due to more failed observations. The detailed description and its advantages will be described in the next section. In this article, we assume that the lifetimes of observations are independent and identically distributed (i.i.d.) with an exponential distribution. Under exponential data assumption, we obtain the exact conditional moment generating function (m.g.f.) of the MLE as well as the exact conditional distributions of the MLE. Also, they are used to obtain the exact conditional confidence interval for the exponential parameter.

The rest of the paper is organized as follows. We consider the case of an exponential distribution under generalized progressive hybrid censored sample and discuss the MLE of the parameters in Section 2. We then derive the exact conditional m.g.f. of the MLE through which we derive the exact conditional distributions of the MLE and use them to obtain exact conditional confidence interval for the parameter in Section 3. A Monte Carlo simulation of inferential procedures is carried out in