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Estimation in step-stress life tests with complementary risks from the exponentiated exponential distribution under time constraint and its applications to UAV data

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ABSTRACT

In accelerated step-stress life tests, the stress levels are allowed to increase at some pre-determined time points such that information on the lifetime parameters can be obtained more quickly than under normal operating conditions. Because there are often multiple causes for the failure of a test unit, such as mechanical or electrical failures, in this article, a step-stress model under time constraint is studied when the lifetimes of different complementary risk factors are independent from exponentiated distributions. Although the baseline distributions can belong to a general class of distributions, including Weibull, Pareto, and Gompertz distributions, particular attention is paid to the case of an exponentiated exponential distribution. Under this setup, the maximum likelihood estimators of the unknown scale and shape parameters of the different causes are derived with the assumption of cumulative damage. Using the asymptotic distributions and the parametric bootstrap method, the confidence intervals for the parameters are then constructed. The precision of the estimates and the performance of the confidence intervals are also assessed through extensive Monte Carlo simulations, and finally, the inference methods discussed here are illustrated with motivating examples.

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1. Introduction

1.1. Step-stress life test with multiple risks

As products become highly reliable with substantially long life-spans, time-consuming and expensive tests are often required to collect a sufficient amount of failure data for analysis. This difficulty is overcome through the use of accelerated life tests (ALT), in which the units are subjected to higher than normal stress levels to induce rapid failures. Among the various stress-loading schemes, the step-stress ALT allows the experimenter to gradually increase the stress levels at some pre-determined time points for maximal flexibility and adjustability. This testing method has attracted considerable attention in the reliability literature. Bagdonavicius [4] and Nelson [28] discussed one of the fundamental models in this area, known as the *cumulative damage* or *cumulative exposure* model. Dharmadhikari and Rahman [12] discussed various parametric models for the step-stress ALT, and Abd-Elfattah et al. [1] studied the estimation problem for a step-stress partial ALT under a Burr-type XII distribution with Type I censoring. Recently, an exact conditional inference for a step-stress model with exponential competing risks was developed by Balakrishnan and Han [5] and Han and Balakrishnan [20]. Gouno, Sen and Balakrishnan [14] and Balakrishnan and Han [6] addressed the problem of determining the optimal stress duration under progressive Type-I censoring; see also Han et al. [21] for some related comments. More recently, Han and Ng [22] quantified the advantage of using the step-stress ALT over the constant-stress ALT under several optimality criteria.

During reliability analyses, it is common for a failure to be associated with one of several risk factors that the test unit is exposed to. Because it is not usually possible to study the test units with an isolated risk factor, it becomes necessary to assess each risk factor in the presence of other risks. To analyze such a multiple risks model, each failure observation must come in a bivariate format composed of a failure time and the corresponding cause of failure. Prentice et al. [29] summarized two approaches for modeling the multiple risks: the cause-specific hazard functions and the latent failure times for each risk factor. Cox [10], Klein and Basu [25,26], and Crowder [11] all investigated the competing risks models with some specific parametric distributions for each risk factor. Basu and Klein [7] introduced an iteration of the *complementary risks* model that has a physical interpretation for describing the lifetime of a parallel system. Motivated by an engineering case study described in Section 6, the complementary risks model is formulated in this paper when the lifetime of each risk factor is from an exponentiated distribution, which is analogous to Lehmann alternatives; see [15].

1.2. Exponentiated exponential distribution

The two-parameter *exponentiated exponential* (EE) or *generalized exponential* (GE) distribution is a special case of an exponentiated distribution, whose cumulative distribution function (CDF) and probability density function (PDF) are given as

$$F_T(t) = \left(1 - e^{-\lambda t}\right)^{\alpha}, \quad t > 0$$
$$f_T(t) = \alpha\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{\alpha-1}, \quad t > 0$$

where $\lambda > 0$ and $\alpha > 0$ are the scale and shape parameters. As a special case, when $\alpha = 1$, the given distribution reduces to a simple exponential distribution with the rate parameter λ. The hazard rate function (HRF) of the EE distribution is defined as

$$h_T(t) = \frac{\alpha\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{\alpha-1}}{1 - \left(1 - e^{-\lambda t}\right)^{\alpha}}, \quad t > 0.$$

Depending on the value of α, this function can be increasing (i.e., $\alpha > 1$), decreasing (i.e., $\alpha < 1$), or constant (i.e., $\alpha = 1$). This strong flexibility makes the EE distribution widely applicable in life tests.

The EE distribution was introduced by Mudholkar and Srivastava [27] as an alternative to the popular Weibull, gamma, and log-normal distributions. Gupta and Kundu [17,18] observed that this distribution can be used quite effectively to analyze many lifetime data, providing a better fit particularly