Statistical Methodology 20 (2014) 2–10

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Kesar Singh's contributions to statistical methodology

G. Jogesh Babu *

Department of Statistics, 326 Thomas Building, The Pennsylvania State University, University Park, PA, 16802, United States

ARTICLE INFO

Article history:
Received 4 June 2013
Accepted 10 December 2013

Keywords:
Bootstrap
Robust inference
Edgeworth expansions
Data depth
Confidence distribution
L-estimators

ABSTRACT

A selection of Professor Kesar Singh's contributions to statistical theory is discussed. The topics reviewed include: Bootstrap methodology, *L*-estimators, robust estimation, Edgeworth expansions, Data Depth and his 'final' contribution which appeared after he left us.

1. Introduction

Professor Kesar Singh, a beloved colleague, friend and vibrant researcher, had a massive heart attack and left us forever, and far too soon, on Wednesday May 16, 2012. Kesar Singh was born on June 20, 1955 into a farming family just outside of Varanasi, India. He got his B.Sc. degree in 1973 from Allahabad University and Ph.D. in 1979 from the Indian Statistical Institute (ISI), Kolkata. He was my first Ph.D. student. His thesis was on asymptotic theory of quantiles and *L*-statistics. Kesar Singh was considered exceptionally brilliant by his teachers and fellow students at ISI. Highlights of his statistical research spanning more than three decades are reviewed here.

2. The early years

Kesar Singh's early work was on *L*-estimators, empirical and quantile processes, probabilities of moderate and large deviations for dependent processes, and Strassen's *r*-quick convergence. His

* Tel.: +1 814 863 2837; fax: +1 814 863 7114.
E-mail address: stamet@stat.psu.edu.

1572-3127/$ – see front matter
http://dx.doi.org/10.1016/j.stamet.2013.12.001

results on these topics were published during 1977–1983 [1–10,13,15]. Some of them are joint work with the author and others.

Kesar Singh's intuition that the location estimators that are linear combinations of order statistics giving less weight to extreme data points are better for distributions having heavier tails paid off. He provided theoretical support for this at various levels of generality by introducing a 'new' condition for heaviness of tail. He indicated a method for calculating relative efficiencies over the class of symmetric unimodal distributions [1]. He established an asymptotic almost sure representation for L-statistics based on i.i.d. observations in [10]. He also provided nonuniform rates of convergence to normality for linear functions of order statistics which are useful in the study of moment convergence.

Kesar Singh along with his co-authors developed asymptotic expressions for tail probabilities of the deviation of the sample mean from the population mean for dependent processes including strong and uniform mixing random variables, and linear processes [3–5,15]. They derived both uniform and nonuniform Berry–Esseen bounds for sums of nonstationary uniform mixing random variables. The bounds are then applied to establish convergence of absolute moments of such sums to the corresponding moments of the standard normal distribution. In [9], he obtains large deviation probabilities for the sample mean $\overline{X}_n$ and for certain functions of the empirical distribution, including Anderson–Darling and Kolmogorov–Smirnov statistics, when the observations $\{X_i\}$ obey the following dependence models: (i) $X_i = \sum_{j=0}^{\infty} a_j Z_{i-j}, i \geq 1$; $\{a_j\}$ real numbers such that $\sum_{j=0}^{\infty} |a_j| < \infty, \sum_{j=0}^{\infty} a_j \neq 0$ and $\{Z_j\}_{-\infty}^{\infty}$ i.i.d. mean zero random variables. (ii) $\{X_i\}$ is an m-dependent, stationary, mean zero sequence of random variables. Various applications and some extensions to the multivariate case are also discussed. He continued his work on large deviation probabilities for non-central t-statistics in [25].

In [2], after developing a number of useful inequalities, Kesar Singh and his co-author estimate the deviations between empirical and quantile processes for mixing random variables with almost sure rates of convergence. The results are similar to those of Bahadur and Kiefer in the i.i.d. case. The results of [2] give the sharpest possible orders in view of the corresponding i.i.d. result of Kiefer. Kesar Singh relaxes considerably the conditions on the mixing rate used in earlier work, and develops an approximation result for the strong mixing case with only polynomially decaying mixing coefficients in [8]. Some of the inequalities developed in [2] are used in [6] by him to approximate a quantile process by an empirical process and provide nonuniform error bounds for dependent observations. In this representation of quantile processes, the order of the remainder gets sharper and sharper as one moves towards sample extremes. More precisely, let $U_1, U_2, \ldots$ be a stationary m dependent sequence of random variables each having a uniform distribution on the interval (0, 1). If F_n is the empirical distribution function and F_n^{-1} is its left continuous inverse, *i.e.*, the sample quantile function, then

$$\sup_{\varepsilon_n \leq t \leq 1-\varepsilon_n} [t(1-t)]^{-1/4} |F_n^{-1}(t) - t + F_n(t) - t| = O\{n^{-3/4}(\log n)^{3/4}\}$$

almost surely, where $\varepsilon_n = n^{-1} \log n$. He also obtains the corresponding result for φ-mixing uniform (0, 1) random variables. This extends a result of [67] by introducing the weight function and weak dependence of the random variables. The key to these and subsequent results is the following inequality developed in [2]: for a strictly stationary uniform mixing process $\{X_n\}$, with uniformly distributed marginals, there exists a $d > 0$ such that, whenever $0 \leq \alpha < 1, 0 < |\beta| \leq b \leq 1 - \alpha, |\beta|, 1 \leq k \leq m, H \geq 0$, and $0 < D \leq bm^{19/24}$, we have

$$P\left(\left|\sum_{i=H+1}^{H+k} I(\min(\alpha, \alpha+\beta)) \leq X_i < \max(\alpha, \alpha+\beta) - |\beta|\right| > 2dD\right) \leq d_1 m^{-4} + d_2 e^{-8D^2/mb},$$

under some mixing conditions, where d_1, d_2 are some constants.

Kesar Singh also contributed to the so called r-quick convergence introduced by [69] as a stronger version of his functional log–log law. A sequence of random elements $\{f_n\}$ in a metric space (M, d) is r-quickly relatively compact with limit set $U \subset M$ if, for each $\varepsilon > 0$, $E(\sup\{n \geq 1 : f_n \notin U_\varepsilon\})^r < \infty$, where $r > 0$, U_ε is the ε-neighborhood of U, and $E(\sup\{n \geq 1 : d(x, f_n) \leq \varepsilon\})^r = \infty$ for each $x \in U$. Kesar Singh and the author, after giving an r-quick version of the Cramér–Wold device, developed asymptotic results for mixing strictly stationary sequences in [13]. The paper also contains the r-quick