Statistical Methodology 18 (2014) 101–114

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Generalized inverted exponential distribution under hybrid censoring

Sanku Dey [a], Biswabrata Pradhan [b,*]

[a] *Department of Statistics, St. Anthony's College, Shillong, Pin-793001, India*
[b] *SQC & OR Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata, Pin 700108, India*

ARTICLE INFO

Article history:
Received 17 November 2012
Received in revised form
11 July 2013
Accepted 11 July 2013

Keywords:
Type-I and type-II censoring schemes
Fisher information matrix
Loss function
Lindley's approximation
Importance sampling
Optimum censoring scheme

ABSTRACT

The hybrid censoring scheme is a mixture of Type-I and Type-II censoring schemes. Based on hybrid censored samples, we first derive the maximum likelihood estimators of the unknown parameters and the expected Fisher's information matrix of the generalized inverted exponential distribution (GIED). Monte Carlo simulations are performed to study the performance of the maximum likelihood estimators. Next we consider Bayes estimation under the squared error loss function. These Bayes estimates are evaluated by applying Lindley's approximation method, the importance sampling procedure and Metropolis–Hastings algorithm. The importance sampling technique is used to compute the highest posterior density credible intervals. Two data sets are analyzed for illustrative purposes. Finally, we discuss a method of obtaining the optimum hybrid censoring scheme.

1. Introduction

The one parameter (negative) exponential distribution is one of the most widely used lifetime models in reliability and survival analysis because of its simple mathematical form and some interesting properties. Gupta and Kundu [13] generalized this model by introducing a shape parameter, which is known as the generalized exponential distribution. Another modification to exponential distribution has been done by using its inverted version, known as the inverted exponential distribution (IED) and was studied by Lin et al. [22]. They obtained the maximum likelihood estimator, confidence limits and UMVUE for the parameter and the reliability function using complete samples. They also compared this model with that of inverted Gaussian and log-normal distributions based on a maintenance data

* Corresponding author.
E-mail address: bis@isical.ac.in (B. Pradhan).

1572-3127/$ – see front matter
http://dx.doi.org/10.1016/j.stamet.2013.07.007

set. Dey [6] obtained Bayes estimators of the parameter and risk functions under different loss functions. While IED has been used for several applications, very little work has been done on the generalized version of the IED. Abouammoh and Alshingiti [1] introduced a shape parameter to the IED to obtain the generalized inverted exponential distribution (GIED). It is to be noted that this distribution originated from the exponentiated Frechet distribution studied by Nadarajah and Kotz [25]. They mentioned that, due to the convenient structure of the distribution function, the generalized inverted exponential distribution (GIED) can be used in many applications, for example in accelerated life testing, horse racing, queue theory, modeling wind speeds etc. The cumulative distribution function (cdf) of GIED with shape parameter α and scale parameter λ is given by

$$F(t;\alpha,\lambda) = 1 - \left(1 - e^{-\lambda/t}\right)^{\alpha}, \quad t > 0, \alpha, \lambda > 0. \tag{1}$$

The corresponding probability density function (pdf) is given by

$$f(t;\alpha,\lambda) = \frac{\alpha\lambda}{t^2} e^{-\lambda/t} \left(1 - e^{-\lambda/t}\right)^{\alpha-1} \quad t > 0, \alpha, \lambda > 0. \tag{2}$$

The reliability function and hazard rate are

$$R(t) = \left(1 - e^{-\lambda/t}\right)^{\alpha}, \quad t > 0, \alpha, \lambda > 0. \tag{3}$$

$$\text{and} \quad h(t;\alpha,\lambda) = \frac{\alpha\lambda}{t^2}(e^{\lambda/t} - 1)^{-1}, \quad t > 0, \alpha, \lambda > 0, \tag{4}$$

respectively.

Several interesting properties of GIED have been studied in detail by Abouammoh and Alshingiti [1] and Nadarajah and Kotz [25]. The hazard rate functions of GIED can be increasing, or decreasing but not constant depending on the value of the shape parameter. The GIED has a unimodal and right skewed density function for the shape parameter greater than 4. Moreover, they observed that in many situations, the GIED provides a better fit than gamma, Weibull, generalized exponential and inverted exponential distributions (see [1]). Recently, Krishna and Kumar [15] studied reliability estimation in the context of this distribution under progressively type II censored sample.

The two most popular censoring schemes found in the reliability literature are Type-I and Type II censoring schemes. The hybrid censoring scheme is a mixture of conventional Type-I and Type-II censoring schemes. The hybrid censoring scheme was first introduced in the literature by Epstein [9,10]. Recently, it has become quite popular in the reliability and life testing experiments. Some early work on hybrid censoring can be found in [11,7,4]. Some recent studies on hybrid censoring have been carried out by many authors including Jeong et al. [14], Gupta and Kundu [12], Childs et al. [5], Kundu [16], Banerjee and Kundu [3], Kundu and Pradhan [18], Dube et al. [8] and Balakrishnan and Kundu [2].

The hybrid censoring scheme can be briefly described as follows. Suppose that n identical units are put on test. The test is terminated when a pre-assigned number r (say), out of n units have failed or a pre-determined time T_0 has been reached. Therefore, in the hybrid censoring scheme, the experimental time and the number of failures will not exceed T_0 and r, respectively. Let $T_1, T_2, \ldots, T_n$ are the lifetimes of the n units. We assume that $T_1, T_2, \ldots, T_n$ are i.i.d. with distribution function (1) and density function (2). Let $T_{1:n} \leq T_{2:n} \leq \cdots \leq T_{n:n}$ be the corresponding order statistic. The number of failures and observation times are denoted by D and $C = \min(T_{r:n}, T_0)$, respectively. Thus, the observed sample is represented by $(T_{1:n}, T_{2:n}, \ldots, T_{D:n}, D)$. Note that when $D = 0$ no failure information is observed.

The objective of the paper is two-fold. The first objective is to obtain the maximum likelihood estimators (MLEs) of the unknown parameters of the model. It is observed that the MLEs can be obtained by solving a single non-linear equation. The observed information matrix is also derived and it is used to obtain the asymptotic confidence intervals for the unknown parameters. We have also obtained Fisher's information matrix emulating Park and Balakrishnan [28]. Next we consider the Bayes estimation of the unknown parameters under the squared error loss function. It is observed that Bayes estimates cannot be obtained in explicit form. We apply Lindley's approximation method, the importance sampling procedure and Metropolis–Hastings algorithm to compute the Bayes estimates