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Multivariate discrete scalar hazard rate

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ABSTRACT

In the present paper, we study the properties of the multivariate discrete scalar hazard rate. Its continuous analogue introduced in the early seventies did not attract much attention because it could not be used to identify the corresponding life distribution. We find the conditions under which an n-variate discrete scalar hazard rate can determine the distribution uniquely. Several other properties of this hazard rate which can be employed in modelling lifetime data are discussed. Some ageing classes based on the scalar hazard function are suggested.

1. Introduction

The concept of hazard rate is fundamental in reliability modelling and analysis. Apart from uniquely determining the lifetime distribution, the hazard rate provides more information than the survival function about the pattern of failure. It also enables the study of various ageing classes that classify life distributions. Further, the distribution of lifetime can be identified using the functional form of the hazard rates through characterizations.

When the hazard rate definition is extended to the multivariate case, there are different ways in which it can be proposed depending on the manner in which the univariate notion is generalized. If (X_1, X_2) is a random vector in the support of N^2, $N = \{0, 1, 2, \ldots\}$ with survival function,

$$S(x_1, x_2) = P[X_1 \geq x_1 X_2 \geq x_2]. \tag{1.1}$$

Nair and Nair [8] defined the bivariate hazard rate as the vector

$$\mathbf{b}(x_1, x_2) = (b_1(x_1, x_2), b_2(x_1, x_2)), \tag{1.2}$$

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where,

$$b_i(x_1, x_2) = P[X_i = x_i | X_1 \geq x_1, X_2 \geq x_2], \quad i = 1, 2. \tag{1.3}$$

Kotz and Johnson [7] proposed

$$\mathbf{c}(x_1, x_2) = (c_1(x_1, x_2), c_2(x_1, x_2)), \tag{1.4}$$

in which

$$c_i(x_1, x_2) = P[X_i = x_i | X_i \geq x_i, X_j = x_j], \quad i, j = 1, 2, \ i \neq j \tag{1.5}$$

as an alternative. In a slightly different framework, Sun and Basu [13] used a three component vector $(d(x), d_1(x_1|x_2), d_2(x_2|x_1))$ in presenting their bivariate total hazard rate with

$$d(x) = \frac{P[\min(X_1, X_2) = x]}{S(x_1, x_2)}, \tag{1.6}$$

$$d_1(x_1|x_2) = \frac{f(x_1, x_2)}{\sum_{t=x_1}^{\infty} f(t, x_2)}, \quad x_1 > x_2 \tag{1.7}$$

and

$$d_2(x_2|x_1) = \frac{f(x_1, x_2)}{\sum_{t=x_2}^{\infty} f(x_1, t)}, \quad x_1 < x_2, \tag{1.8}$$

where, $f(x_1, x_2)$ is the probability mass function of (X_1, X_2). A fourth version of the bivariate hazard rate given by Shaked et al. [12] has five components.

The extension of the above definitions to the n-variate case is straightforward.

The earliest definition of bivariate hazard rate in continuous time, given by Basu [2], is the scalar quantity

$$h(x_1, x_2) = \frac{g(x_1, x_2)}{\bar{G}(x_1, x_2)}, \tag{1.9}$$

where g and $\bar{G}$ respectively denote the density and survival function of (X_1, X_2) of a continuous non-negative random vector. A major limitation of the above definition pointed out by several researchers, e.g. Galambos and Kotz [5], Finkelstein and Esaulova [4] is that $h(x_1, x_2)$ cannot determine the distribution of (X_1, X_2). As a consequence, modelling bivariate data by the functional form of $h(x_1, x_2)$ becomes difficult. Recently, Navarro [9] has obtained some general conditions under which (1.9) determines the corresponding distribution.

In the present work, we study the properties of the discrete n-variate version of (1.9), which does not appear to have been considered earlier. The role of reliability models in discrete time has been well established and the existence of complex equipments with several components whose reliability studies are essential motivates our study. Secondly, we demonstrate that under some mild conditions, the multivariate scalar hazard rate can determine the life distribution, which overcomes the main disadvantage of this concept. In modelling multivariate data, the dependence relation between the constituent variables is of primary importance. The hazard rate considered here can be used to infer the dependence relation that reduces considerably the list of candidate distributions sought for modelling. Identification of appropriate distributions can also be accomplished through some characterizations based simple functional forms of the scalar hazard rates. Certain reservations about the vector representation of hazard functions [3] is not applicable to our results as the hazard rate considered here is a scalar quantity.

The paper is organized into four sections. In Section 2, we define and present the properties of the multivariate discrete scalar hazard rate. This is followed by suggesting how the hazard rate can be employed to study the ageing criteria. Finally in Section 4, we give a short conclusion of the present study.