Statistical Methodology 29 (2016) 51–69

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Combination of mean residual life order with reliability applications

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ARTICLE INFO

Article history:
Received 28 September 2014
Received in revised form
14 September 2015
Accepted 23 October 2015
Available online 31 October 2015

MSC:
62N05
62E10
60K20

Keywords:
Mean residual life
Hazard rate
Characterization
Preservation
Mixture
Shock models
Excess lifetime models
Aging notions
Hypothesis testing applications

ABSTRACT

The purposes of this paper are to introduce a new stochastic order and to study its reliability properties. Some characterizations and preservation properties of the new order under reliability operations of monotone transformation, mixture, weighted distributions and shock models are discussed. In addition, a new class of life distributions is proposed, and some of its reliability properties are investigated. Finally, to illustrate the concepts, some applications in the context of reliability theory and life testing are presented.

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http://dx.doi.org/10.1016/j.stamet.2015.10.001
1572-3127/

1. Introduction and motivations

In the context of reliability and survival analysis, the goal of explicitly calculating a probability distribution is rarely attainable. This makes alternative methods of analysis attractive. One approach is the notion of stochastic orders. Two well-known stochastic orders that have been introduced and studied in reliability theory are the hazard rate (HR) and the mean residual life (MRL) orders (see, Shaked and Shanthikumar [46] and Müller and Stoyan [40] for a broad perspective of the theory and utility of stochastic orderings).

Throughout this paper X and Y are two non-negative random variables with distribution functions F and G, respectively. Denote by f the density function of X and by $\overline{F} = 1 - F$ the corresponding survival function. We use a similar notation for all other distribution functions.

If

$$\bar{G}(t)/\bar{F}(t) \text{ is non-decreasing in } t \geq 0,$$

then X is said to be smaller than Y in the HR order (denoted as $X \leq_{HR} Y$).

If

$$\int_t^\infty \bar{G}(u)du \Big/ \int_t^\infty \bar{F}(u)du \text{ is non-decreasing in } t \geq 0,$$

then X is said to be smaller than Y in the MRL order (denoted by $X \leq_{MRL} Y$).

Another partial ordering, known as combination convexity (CCX) order has been considered and studied by Alzaid [8] and Sekeh et al. [45].

If

$$\int_t^\infty x\bar{F}(x)dx \leq \int_t^\infty x\bar{G}(x)dx, \qquad \text{for all } t \geq 0,$$

then X is said to be smaller than Y in the CCX order (denoted by $X \leq_{CCX} Y$).

In renewal theory, the equilibrium distribution arises as the limiting distribution of the forward recurrence time in a renewal process. Formally, let $\{X_k,\ k = 1, 2, \ldots\}$ be a sequence of mutually independent and identically distributed (i.i.d.) non-negative random variables with common distribution function F. For $n \geq 1$, denote by $S_n = \sum_{i=1}^n X_i$ the time of the nth arrival and let $S_0 = 0$. Let $N(t) = \text{Sup}\,\{n : S_n \leq t\}$ represent the number of arrivals during the interval $[0, t]$. Then, $N = \{N(t),\ t \geq 0\}$ is a renewal process with underlying distribution F (Ross [44]).

Let $\gamma(t)$ be the excess lifetime at time $t \geq 0$, that is, $\gamma(t) = S_{N(t)+1} - t$. Denote by $M(t) = E[N(t)]$ the renewal function which satisfies the following equation

$$M(t) = F(t) + \int_0^t F(t-y)dM(y), \quad t \geq 0.$$

According to Barlow and Proschan [12], for all $t \geq 0$ and $x \geq 0$:

$$P[\gamma(t) > x] = \bar{F}(t+x) + \int_0^t \bar{F}(t+x-u)dM(u).$$

By the elementary renewal theorem, it is straightforward to conclude

$$\begin{aligned}\widetilde{F}(x) &= \lim_{t\to\infty} P(\gamma(t) \leq x)\\ &= \frac{1}{\mu}\int_0^x \bar{F}(u)du, \quad x \geq 0. \end{aligned} \tag{1.1}$$

The distribution function given in (1.1) is the equilibrium distribution of X, where $\mu = E(X) < \infty$ (cf. Abouammoh et al. [2], Abouammoh et al. [3] and Mugdadi and Ahmad [39]). The MRL order is characterized via the HR order as (Hu et al. [21])

$$X \leq_{MRL} Y \Leftrightarrow \widetilde{X} \leq_{HR} \widetilde{Y}. \tag{1.2}$$