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Exact likelihood inference for exponential distributions under generalized progressive hybrid censoring schemes

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ABSTRACT

Generalized Type-I and Type-II hybrid censoring schemes as proposed in Chandrasekar et al. (2004) are extended to progressively Type-II censored data. Using the spacings' based approach due to Cramer and Balakrishnan (2013), we obtain explicit expressions for the density functions of the MLEs. The resulting formulas are given in terms of B-spline functions so that they can be easily and efficiently implemented on a computer.

1. Introduction

In the last decade, various models of hybrid censoring have been proposed (for a recent review, see [5]). Based on truncated life testing as proposed first in [18], Type-I hybrid and Type-II hybrid censored order statistics have been extensively discussed. In particular, likelihood inference for the scale and location parameter of two-parameter exponential distributions $\mathsf{Exp}(\mu, \vartheta)$ with distribution

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and density function as in

$$F(x) = 1 - \mathrm{e}^{-(x-\mu)/\vartheta}, \qquad f(x) = \frac{1}{\vartheta}\mathrm{e}^{-(x-\mu)/\vartheta}, \quad \mu \le x, \ \vartheta > 0,$$

have received great attention and the exact distribution of the MLEs has been addressed in many articles. Starting with Chen and Bhattacharyya [9], the method of conditional moment generating function has been successively applied to derive the density function of the MLE of ϑ in the scale model (for a simplified version, see also [12]). Childs et al. [12] also proposed Type-II hybrid censoring and established respective results for the distribution of the MLE. Since both hybrid censoring procedures face some drawbacks, Chandrasekar et al. [8] proposed extensions called *generalized Type-I and Type-II hybrid censoring schemes* which alleviate some negative effects (see also [5, Section 4]). The model has been further discussed w.r.t. competing risks [24,21] and step-stress models [27]. The Fisher information in generalized hybrid censoring schemes has been addressed in [25]. Further extensions called *unified hybrid censoring scheme* and *flexible hybrid censoring scheme* have been proposed in [6] and in [26], respectively. Initiated by Childs et al. [11] and Kundu and Joarder [23], some of the above models have also been discussed in the model of progressive censoring (see also [2, Chapters 5 and 14]). Similar results for the exact (conditional) distribution of the MLEs have been obtained in these papers as well as in [10,19]. Although these models exhibit more complicated decision rules for generating the data, the method of generating function worked and yielded explicit expressions for the density and distribution functions of the MLEs in the exponential case. However, the resulting expressions were complicated due to alternating sums and truncated gamma functions, and, thus, difficult to implement on a computer.

In this paper, we extend the generalized Type-I and Type-II hybrid censoring schemes to progressively censored data. It has to be noticed that generalized Type-I progressive hybrid censoring has already been discussed recently in [13] whereas the Type-II version is new and has not been addressed so far. In contrast to the moment generating function method applied in [13], we apply the spacings' based approach due to Cramer and Balakrishnan [14] for Type-I progressive hybrid censored data to derive rather compact expressions for the density functions of the MLE of the scale parameter. It should be noted that the same method has recently be successfully applied to Type-II progressively hybrid censored order statistics by Cramer et al. [15]. In comparison to the representations obtained by the moment generating function approach, the resulting expressions in terms of B-splines can be easily and efficiently implemented on a computer. Moreover, they give some insight into the structure of the distributions. Furthermore, it is worth noting that the structure of the formulas remains the same when the procedure is based on Type-II censored data only. Hence, there is no significant simplification in this (simpler) setting.

The present paper is structured as follows. In Section 2, we introduce *generalized Type-I and Type-II progressive hybrid censoring* and present the MLEs for the location and scale parameters for two-parameter exponential distributions. Using these results, we establish in Sections 3 and 4 expressions for the density functions of the MLEs. We discuss both the case of a known and unknown location-parameter μ. In the latter case, we derive the joint density function of the bivariate MLE $(\hat{\mu}, \hat{\vartheta})$. The methods proceed by volume computations of intersections of simplices with half-spaces as already used in [14,15], respectively.

2. Models and likelihood inference

In order to present the new censoring schemes extending the models presented in [8] to progressively Type-II censored data, we first present some basic results. For an exponentially distributed i.i.d. sample $X_1, \ldots, X_n \sim \mathrm{Exp}(\mu, \vartheta)$, a fixed value m, with $m \le n$, and for a prefixed censoring scheme $\mathscr{R} = (R_1, \ldots, R_m)$, we consider a progressively Type-II censored lifetest leading to the data

$$X_{1:m:n}, \ldots, X_{m:m:n}. \tag{2.1}$$

In such a lifetest, successively failure times are observed but at the jth failure time R_j units are randomly chosen among the operating units and removed from the lifetest. Thus, at each failure time,