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Further results on closure properties of LPQE order

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ABSTRACT

Di Crescenzo and Longobardi (2002) introduced the past entropy, Sunoj et al. (2013) gave a quantile version for the past entropy, termed as the past quantile entropy (PQE). Based on the PQE function, they defined a new stochastic order called as less PQE (LPQE) order and studied some properties of this order. In the present paper, we focus our interests on further closure properties of this new order. Some characterizations of the LPQE order are investigated, closure and reversed closure properties are obtained. The preservation of the LPQE order in the proportional failure rate and reversed failure rate models is discussed.

1. Introduction and preliminaries

Let X be a nonnegative absolutely continuous random variable representing the random lifetime of a unit or a system. Assume that X has the probability density function $f_X(x)$, distribution function $F_X(x)$, survival function $\bar{F}_X(x) = 1 - F_X(x)$, and right continuous inverse function F_X^{-1} of F_X defined by $F_X^{-1}(u) = \sup\{x|F_X(x) \leq u\}$, for all $u \in [0, 1]$. A classical measure of the uncertainty contained in X is the differential entropy, also known as Shannon's information measure, defined by

$$H_X = -\mathrm{E}[\ln f_X(X)] = -\int_0^{+\infty} f_X(x) \ln f_X(x) \mathrm{d}x. \tag{1}$$

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See Shannon [31]. This is a measure of the uncertainty of the lifetime of a unit or a system. The idea is that a unit with great uncertainty is less reliable than a unit with low uncertainty.

Since the classical contributions by Shannon [31] and Wiener [35], the properties of H_X have been thoroughly investigated. Subsequently, Ebrahimi and Pellerey [12], Ebrahimi [10], Ebrahimi and Kirmani [11], Di Crescenzo and Longobardi [6], and Navarro et al. [26] investigated this differential entropy.

As suitable generalizations or modifications of the classical Shannon differential entropy, other measures of uncertainty have been proposed in the literature. Di Crescenzo and Longobardi [8] defined the weighted entropy as

$$H_X^w = H^w(X) = -\mathrm{E}[X \ln f_X(X)] = -\int_0^{+\infty} x f_X(x) \ln f_X(x) \mathrm{d}x.$$

Rao et al. [27] defined the cumulative residual entropy as

$$\mathcal{E}_X = \mathcal{E}(X) = -\int_0^{+\infty} \overline{F}_X(x) \ln \overline{F}_X(x) \mathrm{d}x.$$

Di Crescenzo and Longobardi [9] defined the cumulative past entropy as

$$\mathcal{CE}_X = \mathcal{CE}(X) = -\int_0^{+\infty} F_X(x) \ln F_X(x) \mathrm{d}x.$$

Khinchin [16] defined the measure

$$\int_0^{+\infty} f_X(x) \phi(f_X(x)) \mathrm{d}x,$$

and generalized (1) by choosing a convex function ϕ such that $\phi(1) = 0$.

Moreover, Renyi [28] defined the Renyi entropy of order α, Zografos and Nadarajah [36] introduced the survival exponential entropy. Abbasnejad et al. [1] defined the survival entropy of order α, and considered the relationship between the survival exponential entropy and the survival entropy. These entropies are all measures of uncertainty of life random variables, and they can be viewed as different generalizations of the Shannon differential entropy.

By denoting $q_X(u) = \frac{\mathrm{d}F_X^{-1}(u)}{\mathrm{d}u} = \left[f_X(F_X^{-1}(u))\right]^{-1}$, Sunoj and Sankaran [32] gave H_X a quite simple expression

$$H_X = -\int_0^1 \ln\left(f_X[F_X^{-1}(u)]\right) \mathrm{d}u = \int_0^1 \ln\left[q_X(u)\right] \mathrm{d}u. \tag{2}$$

Shannon's information entropy H_X gives a measure of the uncertainty of X. However, this is a static one. The uncertainty is perhaps related to the time point at which it is studied. Based on such an idea, Ebrahimi and Pellerey [12] introduced the residual entropy of X at time $t \geq 0$ by

$$H_X(t) = -\int_t^{+\infty} \frac{f_X(x)}{\overline{F}_X(t)} \ln\left[\frac{f_X(x)}{\overline{F}_X(t)}\right] \mathrm{d}x, \quad \text{for all } t > 0. \tag{3}$$

$H_X(t)$ is a function of age t of X, hence this is a dynamic measure of uncertainty for X. From (1) it can be seen that $H_X(t) = H_{X_t}$, where $X_t = (X - t \mid X > t)$ is the residual life of X at age $t \geq 0$.

Navarro et al. [26], Asadi and Ebrahimi [2], and Belzunce [5] studied the residual entropy. Moreover, numerous dynamic generalizations of (1) have been proposed (see, for instance, Taneja [34], Di Crescenzo and Longobardi [9], Nanda and Paul [25], Gupta and Nanda [13], Khinchin [16], Asadi and Zohrevand [3], Kumar and Taneja [18], and Khorashadizadeh et al. [17]).

However, in many realistic situations, the uncertainty of X is related to the past lifetime rather than the residual lifetime. Motivated by this, Di Crescenzo and Longobardi [6] introduced the past