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Moment inequalities for HNBRUE with hypothesis testing application

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ABSTRACT

The paper deals with life distributions which are harmonic new better than renewal used in expectation (HNBRUE). The goal is to derive moment inequalities and use them for further analysis of the class HNBRUE. We establish new characterization of exponentiality versus HNBRUE. Pitman's asymptotic relative efficiency is employed to assess the performance of the proposed test with respect to other available tests. Finally, we carried out numerical simulation to produce table for the critical values of the test.

1. Introduction

Suppose that the lifetime of a system is described by a nonnegative random variable X with distribution function $F = \{F(x), x \in [0; 1)\}$. In this case we write $X \sim F$ and denote the reliability function by $\bar{F} = 1 - F$, so $\bar{F}(0) = 1$. The residual life of the system under operation at a certain time can be expressed by the equilibrium (or stationary renewal) distribution. To see this, consider a system with a component in operation whose life distribution is F. As soon as the component fails, it is replaced consecutively by independent components which are identically distributed with the same F. In the long run, the residual life distribution of a component under operation at time x is given by the distribution $F_e = \{F_e(x), t \in [0; 1)\}$, where

$$F_e(x) = \frac{1}{\mu}\int_0^x \bar{F}(u)\mathrm{d}u, \quad \text{for } x \geq 0. \tag{1.1}$$

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F_e is called the equilibrium distribution corresponding to the original F; $\mu = \mathbf{E}[X]$ is the mean life to the failure. We write $X_e \sim F_e$ where X_e is called the equilibrium random variable life. It is worth mentioning that the equilibrium distribution F_e behaves like any other life distribution. Therefore, we can study its characteristics using the failure rate function $r(x), x \geq 0$ or the mean residual life function $\mu(x), x \geq 0$ or any other characteristics used in the study of ageing properties.

The residual lifetime of the component is characterized by a random variable X_t associated with the original lifetime X. We can also define $X_e(t)$ to be the residual equilibrium lifetime associated with the X_e. Assuming that the second moment of the distribution F is finite, we can compare these random variables and define some ageing classes. For example, the relation $\mathbf{E}[X_t] \leq \mathbf{E}[X]$ defines the class of NBUE, and $\mathbf{E}[X_e(t)] \leq \mathbf{E}[X]$ defines the class of NBRUE. Other possible comparisons between these random variables have been studied by several authors including, e.g. Shaked and Shanthikumar [19], Müller and Stoyan [13] and Bhattacharjee et al. [5]. Cheng and Lam [6] studied upper and lower bounds for the reliability function in harmonic new better than used in expectation (HNBUE) life distribution class with known first two moments. Pandita and Anuradhab [15] introduced and investigated the testing of new better than used of specified age. Elkahlout [7] studied the relations of HNBRUE class with other existing classes of life distributions. Ahmad [2] used moment inequalities to produce new tests for the classes IFR, NBU, NBUE and HNBUE. Abu-Youssef [1] derived the class a moment inequality for the class NBRUE. For further moment inequalities for ageing classes see Ahmad and Mugdadi [3], Mugdadi and Ahmad [12] and Al-Zahrani and Stoyanov [4]. In this paper, we will use a similar methodology to obtain moment inequalities and construct a new statistics for testing exponentiality versus the class HNBRUE.

2. Moment inequalities

It is observed that comparing the equilibrium (or renewal) classes with existing classes of original distributions are useful in maintenance and repair policies. Let us start with a life distribution F with a finite second order moment, i.e., $m_2 = \int_0^\infty x^2 \mathrm{d}F(x) < \infty$.

Definition 2.1. A life distribution $F = \{F(x), x \in [0, \infty), F(0) = 0\}$ and its reliability function $\bar{F}$ are said to be harmonic new better than renewal used in expectation, if

$$\int_x^\infty \int_y^\infty \bar{F}(u)\mathrm{d}u\mathrm{d}y \leq \mathrm{e}^{-x/\mu} \int_0^\infty \int_y^\infty \bar{F}(u)\mathrm{d}u\mathrm{d}y, \quad \text{for all } x \geq 0. \tag{2.1}$$

We use the easy notation **H** for this class instead of the traditional HNBRUE.

The corresponding concept of harmonic new worse renewal than used in expectation class is defined by reversing the inequality sign in (2.1).

Comment. It is important to mention that in relation (2.1) equality holds if and only if the distribution F is $\mathrm{Exp}(\mu)$, i.e., the life distribution of the system is approaching a stable regime which is described by the exponential distribution; see e.g. Bhattacharjee et al. [5]. Later we make use of such a property and build new test statistic for exponentiality.

The meaning of the class **H** can be seen if we rewrite (2.1) in the following equivalent form:

$$\frac{1}{x}\int_0^x \frac{1}{\mu_e(u)}\,\mathrm{d}u \leq \frac{1}{\mu}.$$

Functions as that one in the left-hand side, which is an integral average, are used in the theory of harmonic functions. Because of this specialists in Reliability analysis adopted the name 'harmonic' type of life distributions. For us, the left-hand side is the average time to failure up to time x of the equilibrium system corresponding to F. Hence, relation (2.1) tells us that we require this average time, for any $x > 0$, to be not larger than the average time $1/\mu$ of the exponential distribution with parameter μ.

For our purposes we redefine (2.1) in the following equivalent form:

$$V(x) \leq \mathrm{e}^{-x/m_1} V(0) = \frac{1}{2}\, m_2\, \mathrm{e}^{-x/m_1}, \quad x \geq 0, \tag{2.2}$$