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Empirical likelihood approach toward discriminant analysis for dynamics of stable processes

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A R T I C L E I N F O

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A B S T R A C T

Discriminant analysis for time series models has been studied by many authors in these few decades, but many of them deal with second order stationary processes. In this paper, we introduce an empirical likelihood statistic based on a Whittle likelihood as a classification statistic, and consider problems of classifying an α-stable linear process into one of two categories described by pivotal quantities $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ of time series models. It is shown that misclassification probabilities by the empirical likelihood criterion converge to 0 asymptotically without assuming that the true model is known. We also evaluate misclassification probabilities when $\boldsymbol{\theta}_2$ is contiguous to $\boldsymbol{\theta}_1$, and carry out simulation studies to make a comparison between goodness of the empirical likelihood classification statistic and that of an existing method. We observed that the empirical likelihood ratio discriminant statistic performs better than the existing method in some cases even if a family of score functions does not contain the true model. Since the stable processes do not have the finite second moment, this extension is not straightforward, and contains a lot of innovative aspects.

1. Introduction

In the last few decades, heavy-tailed data have been observed in variety of fields involving electrical engineering, hydrology, finance and physical systems. When we estimate the tail index of the financial

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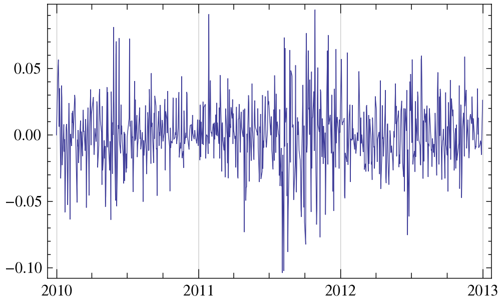
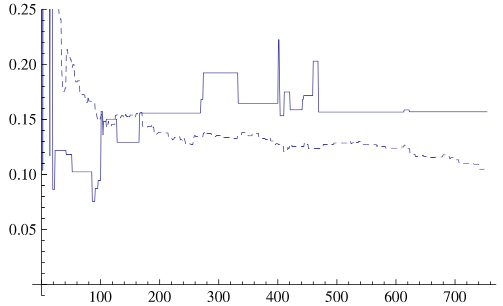


(a) Log returns of Hewlett Packard's stock prices (from 4, January, 2010 to 31, December, 2012).



(b) Hill-plot for the log returns (dashed line is for i.i.d. normal random variables).

Fig. 1. Log returns of Hewlett Packard company and the Hill-plot.

data by Hill-plot, we often observe that the tail of the distribution is heavier than that of Gaussian one. Fig. 1 shows the daily log stock returns of Hewlett Packard company $X(1), \ldots, X(753)$ (Fig. 1(a)), and its Hill-plot (Fig. 1(b)). The right graph implies that the Gaussian model poorly grasps the feature of the data (see Drees, De Haan and Resnick [3], Hill [5], Hall [4] and Hsing [6]). To model such data suitably, in this paper we consider the following linear process

$$X(t) = \sum_{j=0}^{\infty} \psi_j Z(t-j), \quad t \in \mathbb{Z}, \tag{1.1}$$

called a symmetric α-stable linear process, where $\{Z(t); t \in \mathbb{Z}\}$ are independent and identically distributed (i.i.d.) symmetric α-stable random variables with characteristic function $\mathrm{E}[\exp\{iuZ(1)\}] = \exp\{-\sigma|u|^{\alpha}\}$, $\sigma > 0$ and $\mathbb{Z}$ denotes the set of all integers. If α is less than 2, the process has neither the second moment nor the usual spectral density function. A series of works for the stable processes have been done by several authors. In the stable case, Davis and Resnick [2] showed that sample autocovariance functions converge to some stable random variables, and the rate of convergence is greater than that of the second order stationery cases. Periodogram and its normalized version are also fundamental tool for the stable processes. Klüppelberg and Mikosch [8] studied convergence of the "*self-normalized periodogram*"

$$\widetilde{I}_{n,X}(\omega) = \frac{\left|\sum_{t=1}^{n} X(t)\exp(it\omega)\right|^2}{\sum_{t=1}^{n} X(t)^2} \tag{1.2}$$