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On properties of progressively Type-II censored order statistics arising from dependent and non-identical random variables

M. Rezapour [a], M.H. Alamatsaz [b], N. Balakrishnan [c], E. Cramer [d,*]

[a] Department of Statistics, Shahid Bahonar University, Kerman, Iran
[b] Department of Statistics, University of Isfahan, Isfahan, 81746-73441, Iran
[c] Department of Mathematics and Statistics, McMaster University, Ontario L8S 4K1, Canada
[d] Institute of Statistics, RWTH Aachen University, Aachen, Germany

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ABSTRACT

In this paper, we study progressively Type-II censored order statistics arising from identical as well as non-identical units under test which are jointly distributed according to an Archimedean copula with completely monotone generator (PCOSDNARCM-II). Density, distribution and joint density functions of PCOSDNARCM-II are all derived. For certain special cases, more explicit expressions are presented. Some interesting recurrence relations and transformational properties are also established. Results established here contain the results by Balakrishnan and Cramer [5] as particular cases. Finally, some examples of PCOSDNARCM-II are also provided.

1. Introduction

An efficient way of collecting lifetime data that results in a saving in experimental time and cost is progressive Type-II censoring. When the units under test are dependent, we should use a model that takes this dependency into account. Let us consider a dependent progressively Type-II censored sample. Under this scheme, N dependent units (with some joint distribution) are placed on a life-test; after the i-th failure, $R_i (i = 1, \ldots, m \leq N)$ surviving units are removed at random from the test.

* Corresponding author.
E-mail addresses: mohsenrzp@gmail.com (M. Rezapour), alamatho@sci.ui.ac.ir (M.H. Alamatsaz), bala@mcmaster.ca (N. Balakrishnan), erhard.cramer@rwth-aachen.de (E. Cramer).

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Thus, under the dependent progressive Type-II censored sampling, we observe in all m failures, so that $N = m + R_1 + R_2 + \cdots + R_m$. Many authors have studied progressive Type-II censoring and properties of order statistics arising from such a progressively censored life-test. Some key references are [1–4,6,9–14,27]. In all these works, the classical model, as introduced by Herd [19] and Cohen [13], is investigated. Under this model, the lifetimes $X_1, \ldots, X_N$ are supposed to be independent and identically distributed (IID). Recently, some efforts have been made to weaken this assumption. Balakrishnan and Cramer [5] considered progressively Type-II censored order statistics (PCOS-II) arising from independent, but not necessarily identically distributed random variables. They established some basic distributional results and a relation to permanents in this case. Some additional results in this direction are presented in [16]. In [18], a general mixture representation has been obtained which relates the distribution of PCOS-II from an arbitrary sample $X_1, \ldots, X_N$ to the distribution of order statistics from this sample. The identity holds without any restrictions on either the dependence structure or the distributions of the lifetimes. The notion of dependence in the context of progressive Type-II censoring has been discussed in [28], but with the marginal distributions being assumed to be the same. In this work, we extend this model to allow for both dependence and non-identical distributions. In particular, the dependence in the sample $X_1, \ldots, X_N$ is modeled by assuming a copula function for the joint distribution of the lifetimes.

In real lifetime systems, it is quite common for the components to be dependent. Copulas are quite useful to model dependence structures of variables, and have been widely used in modeling dependence in many areas and, especially, in reliability and survival analysis. It is, therefore, natural to apply certain copulas to model the dependency of units under test. In fact, [28,30,29] have recently applied Archimedean copulas to model systems with dependent components. In this paper, we shall follow the same idea but by relaxing the condition of independent and identically distributed life times for the progressively Type-II censored order statistics and then study their distributional properties. It is also of interest to mention here that Eryilmaz [17] has modeled dependencies of components in reliability systems by assuming exchangeability of the corresponding component lifetimes through the copula approach. Jia et al. [20] have assumed copula models for the lifetimes in consecutive k-out-of-n: G systems. The connection to order statistics has been discussed in [25]. In [31], some applications of copulas in the study of order statistics and coherent systems have been reviewed. A convenient subclass of copulas is formed by the class of Archimedean copulas, which has a close connection to Laplace transforms. For a thorough discussion on Archimedean copulas, we refer the interested reader to [21,26].

As mentioned above, the mixture representation of the distribution of PCOS-II obtained in [18] yields an expression for computing the distribution and density functions of PCOS-II in a very general setting provided the corresponding expressions for distributions of order statistics are available. However, this result possesses all the disadvantages in a usual mixture representation, such as the number of terms in the sum becoming large. On the other hand, simple expressions for the (joint) density functions are available in the IID setting. In this work, therefore, we aim to find a setting for dependent lifetimes leading to tractable expressions for the distributions. It turns out that such a setting is provided by the Archimedean copulas. In Section 2, we obtain the density and distribution functions of progressively Type-II censored order statistics arising from dependent and non-identical random variables when the joint distribution of the units under test follows an Archimedean copula. The main result (viz., Theorem 2.1) shows that the joint density function can be expressed as a continuous mixture of distributions of PCOS-II from independent PCOS-II. The marginal density and distribution functions of the r-th PCOS-II as well as the joint density function of the r-th and s-th PCOS-II arising from N identical but dependent random variables following an Archimedean copula with completely monotone generator (PCOSDNARCM-II) are obtained in Section 3. In this section, we also present some recurrence relations for the joint distributions of interest and discuss certain transformational properties. An explicit form for the marginal density and distribution function of the r-th PCOSDNARCM-II as well as the joint density function of the r-th and s-th PCOSDNARCM-II in a specific case are derived in Section 4. We find that the one-dimensional marginal distributions exhibit a structure similar to the IID case (see [23]). Finally, we provide some examples of PCOSDNARCM-II for illustrative purposes.