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A note on concomitants of records

Arup Bose, Sreela Gangopadhyay *

Statistics and Mathematics Unit, Indian Statistical Institute, Kolkata, India

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ABSTRACT

We study the asymptotic distribution properties of concomitants of records and Pfeifer records and provide some interesting examples on the different possible behaviour of the limits. We also show that under suitable conditions, the unnormalized partial sum of concomitants of lower and upper records converges in distribution and the limit is infinitely divisible.

1. Introduction

Study of records was initiated by Chandler [9] and contributions from numerous researchers have enriched the theory of records and its ramifications. See [1] for a comprehensive bibliography. An associated field of interest is the study of concomitants of records defined as follows in the i.i.d. setup. Let $\{(X_n, Y_n)\}_{n\geq 1}$ be a sequence of i.i.d. bivariate random variables, with a common joint cdf $F(x, y)$. We assume that F is continuous. Let us consider $\{X_n\}$, the i.i.d. sequence of random variables with common cdf F_X, which is the marginal of F. We define the sequence of records for $\{X_n\}$ in the usual way as follows.

Let $L_0 = 1$. Suppose L_i are defined for all $i < n$. Then define $L_n = \inf\{k > L_{n-1} : X_k > X_{L_{n-1}}\}$. Define $R_n = X_{L_n}$. Then $\{R_n\}$ is the sequence of upper records of $\{X_n\}$. We denote the corresponding Y-coordinates, Y_{L_n} by $R_{[n]}$ and the sequence $\{R_{[n]}\}$ is called the concomitants of upper records. When there is no chance of confusion we simply refer to them as concomitants of records. We may define lower records of X_n and concomitants of lower records in an analogous way.

Concomitants of records can arise in several applications. Suppose individuals are to be selected on the basis of measurement of an attribute **A** whose high value is desirable. Suppose **B** is an associated

* Corresponding author. Tel.: +91 9830544885.
E-mail addresses: bosearu@gmail.com (A. Bose), gangopadhyay7@gmail.com (S. Gangopadhyay).

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attribute which is known to be positively (or negatively) correlated with **A**. While **B** is easy to measure, **A** is not. So the individuals are first measured on the basis of their **B** values and only those having **B** value bigger (or smaller) than all previous observations on **B** qualify to be measured for their **A** values. A sequence of **A** values thus measured are concomitants of records. One can think of many such examples where concomitants of records are useful.

Concomitants of order statistics attracted a considerable amount of attention in the literature (see for example, [3,4,10,11,18] etc.) but concomitants of records received comparatively little attention. Some important references are Chacko and Thomas [8], and Houchens [13]. In this note we will discuss some interesting results on asymptotic behaviour of concomitants of records. In Section 2.1, we derive a relation between the limiting distribution of normalized records and that of concomitants. In Section 2.2 we consider the concomitants of Pfeifer records (see [15]). In Section 3, we present the asymptotic behaviour of sums of concomitants under certain conditions. Except for Section 2.2, everywhere else we assume $\{(X_i, Y_i)\}$ to be a sequence of i.i.d. bivariate random variables.

2. Main results

Our basic assumption is that the records sequence itself has a nondegenerate limit distribution. For conditions under which this happens, see [16,17] or [1].

Assumption I. There exist sequences of reals $\{a_n\}$ and $\{b_n\}$ and a nondegenerate random variable R such that

$$\frac{R_n - b_n}{a_n} \xrightarrow{\mathcal{D}} R \quad \text{as } n \to \infty. \tag{2.1}$$

Throughout this article, $F_{Y|X}$ denotes the conditional cdf of Y given X.

The following result is an analogue of a result for concomitants of order statistics (see [10]), and does not seem to be available in the literature.

Theorem 1. *Suppose $\{(X_i, Y_i)\}$ are i.i.d. and $\{R_n\}$ is the record sequence of the X variables. Suppose Assumption I holds. If for some c_n and d_n and any fixed y,*

$$F_{Y|X=a_n r+b_n}(c_n y + d_n) \to G(r, y) \quad \text{as } n \to \infty \tag{2.2}$$

locally uniformly in r, where $G(r, y)$ is continuous in r, then

$$\mathbb{P}\left(\frac{R_{[n]} - d_n}{c_n} \leq y\right) \to \int_{-\infty}^{\infty} G(r, y) F_R(dr).$$

Proof. The proof is easy and similar to that available for concomitants of order statistics. For completeness we give it here.

$$\begin{aligned}\mathbb{P}(R_{[n]} \leq d_n + c_n y) &= \int_{-\infty}^{\infty} \mathbb{P}(R_{[n]} \leq d_n + c_n y | R_n = x) F_{R_n}(dx) \\ &= \int_{-\infty}^{\infty} \mathbb{P}(Y_{L_n} \leq d_n + c_n y | X_{L_n} = x) F_{R_n}(dx). \end{aligned}\tag{2.3}$$

Since (X_i, Y_i) are i.i.d., by Lemma 1 in the Appendix,

$$\mathbb{P}(Y_{L_n} \leq d_n + c_n y | X_{L_n} = x) = F_{Y|X=x}(d_n + c_n y).$$

Now putting $x = a_n r + b_n$, we have

$$\mathbb{P}(R_{[n]} \leq d_n + c_n y) = \int_{-\infty}^{\infty} F_{Y|X=a_n r+b_n}(d_n + c_n y) F_{\frac{R_n - b_n}{a_n}}(dr).$$