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A note on Parisian ruin with an ultimate bankruptcy level for Lévy insurance risk processes

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ABSTRACT

In this short paper, we investigate a definition of Parisian ruin introduced in Czarna (2016), namely Parisian ruin with an ultimate bankruptcy level. We improve the results originally obtained and, moreover, we compute more general Parisian fluctuation identities.

1. Introduction

In classical ruin theory, the company is ruined when the surplus process falls below a critical threshold level. Inspired by Parisian options (see e.g. Chesney et al., 1997), some insurance risk models now consider the application of an implementation delay in the recognition of an insurer's capital insufficiency. More precisely, it is assumed that Parisian ruin occurs if the excursion below the critical threshold level is *too long*. The idea stems from the observation that in some industries, companies can continue to do business even though their wealth process falls below the critical level; see Landriault et al. (2014) for more motivation.

The idea of Parisian ruin has generated two types of models: with a deterministic implementation delay or a stochastic delay. The model with a deterministic delay has been studied in the Lévy setup by Czarna and Palmowski (2011), Loeffen et al. (2013) and more recently by Czarna (2016), while Landriault et al. (2011, 2014) and Baurdoux et al. (in press) have considered the idea of Parisian ruin with a stochastic implementation delay, with an emphasis on exponentially distributed delays.

In this paper, we study a general Lévy insurance risk model subject to Parisian ruin with an ultimate bankruptcy barrier, as defined in Czarna (2016). After calculating the probability of this type of Parisian ruin, we will derive a Parisian extension of the two-sided exit problem.

1.1. Lévy insurance risk processes

Let $X = \{X_t, t \geq 0\}$ be a spectrally negative Lévy process (SNLP) on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t, t \geq 0\}, \mathbb{P})$, that is a process with stationary and independent increments and no positive jumps. We exclude the case that X is

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the negative of a subordinator, i.e. we exclude the case of X having decreasing paths. In the actuarial ruin theory literature, processes as X are known as Lévy insurance risk processes. Note that the Cramér–Lundberg risk process and the Brownian approximation risk process belong to this family of stochastic processes. For more on the use of SNLPs in actuarial ruin theory, see e.g. Klüppelberg and Kyprianou (2006) and Kyprianou (2013, 2014).

The law of X such that $X_0 = x$ is denoted by $\mathbb{P}_x$ and the corresponding expectation by $\mathbb{E}_x$. We write $\mathbb{P}$ and $\mathbb{E}$ when $X_0 = 0$. The Laplace transform of X is given by

$$\mathbb{E}\left[e^{\theta X_t}\right] = e^{t\psi(\theta)},$$

for $\theta \geq 0$, where

$$\psi(\theta) = \gamma\theta + \frac{1}{2}\sigma^2\theta^2 + \int_0^\infty \left(e^{-\theta z} - 1 + \theta z \mathbf{1}_{(0,1]}(z)\right) \Pi(dz),$$

for $\gamma \in \mathbb{R}$, $\sigma \geq 0$, and where Π is a measure on $(0, \infty)$ called the Lévy measure of X and such that

$$\int_0^\infty (1 \wedge z^2)\Pi(dz) < \infty.$$

Even though X has only negative jumps, for convenience we choose the Lévy measure to have only mass on the positive instead of the negative half line. Further, note that the *net profit condition* can be expressed as $\mathbb{E}[X_1] = \psi'(0+) > 0$. Note also that the process X has paths of bounded variation if and only if $\sigma = 0$ and $\int_0^1 z\Pi(dz) < \infty$; this is the case when X is a Cramér–Lundberg process since then $\Pi(dz) = \lambda F(dz)$ where λ is the jump/claim rate of the underlying Poisson process and $F(dz)$ is the jump/claim distribution.

1.2. The idea of Parisian ruin

Parisian ruin occurs if the excursion below the critical threshold level 0 is longer than a deterministic time called the implementation delay or the clock. It is worth pointing out that this definition of ruin is referred to as Parisian ruin due to its ties with Parisian options; see Chesney et al. (1997). In Czarna and Palmowski (2011) and Loeffen et al. (2013), a Parisian ruin time (with delay $r > 0$) is defined as

$$\kappa_r = \inf\{t > 0 : t - g_t > r\},$$

where $g_t = \sup\{0 \leq s \leq t : X_s \geq 0\}$. In other words, the company is said to be *Parisian ruined* if there exists an excursion below zero longer than the fixed implementation delay r. Therefore, $\mathbb{P}_x(\kappa_r < \infty)$ is the probability of Parisian ruin, when the initial capital is x, for which a nice and compact expression was obtained in Loeffen et al. (2013): if $\mathbb{E}[X_1] > 0$, then

$$\mathbb{P}_x(\kappa_r < \infty) = \begin{cases} 1 - \mathbb{E}[X_1]\dfrac{\int_0^\infty zW(x+z)\mathbb{P}(X_r \in dz)}{\int_0^\infty z\mathbb{P}(X_r \in dz)} & \text{for } x \geq 0, \\ 1 - \mathbb{E}[X_1]\dfrac{\mathbb{P}_x\left(\tau_0^+ \leq r\right)}{\int_0^\infty z\mathbb{P}(X_r \in dz)} & \text{for } x < 0, \end{cases}$$

where W is the so-called 0-scale function of X (see the definition below) and τ_0^+ is the first passage time above 0.

Later in Czarna (2016), a Parisian ruin time with a lower ultimate bankruptcy level was proposed. In this case, if the excursion below 0 is *too deep*, namely if the surplus goes below level $-a$, then even if the clock has not rung ruin is declared. For this more general stopping time, we first fix $a > 0$ and then define the Parisian ruin time with ultimate bankruptcy level $-a$ as

$$\kappa_r^a := \kappa_r \wedge \tau_{-a}^- = \min(\kappa_r, \tau_{-a}^-),$$

where τ_{-a}^- is the first passage time below $-a$. For this definition of Parisian ruin, a probabilistic decomposition was obtained in Czarna (2016) for $\mathbb{P}_x\left(\kappa_r^a < \infty\right)$ and expressed in terms of the scale functions and the Lévy measure of X.

1.3. Scale functions and fluctuation identities

For an arbitrary SNLP with Laplace exponent ψ, there exists a function $\Phi : [0, \infty) \to [0, \infty)$ defined by $\Phi(q) = \sup\{\theta \geq 0 \mid \psi(\theta) = q\}$ such that $\psi(\Phi(q)) = q$. Note that we have $\Phi(q) = 0$ if and only if $q = 0$ and $\mathbb{E}[X_1] = \psi'(0+) \geq 0$.

We now recall the definition of the q-scale function $W^{(q)}$. For $q \geq 0$, the q-scale function of the process X is such that $W^{(q)}(x) = 0$ for all $x < 0$ and is the unique continuous function on $[0, \infty)$ with Laplace transform given by

$$\int_0^\infty e^{-\theta y} W^{(q)}(y) dy = \frac{1}{\psi(\theta) - q}, \quad \text{for } \theta > \Phi(q), \tag{1}$$