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Rank test statistics for unbalanced nested designs

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Abstract

We formulate rank statistics for testing hypotheses in unbalanced, and possibly heteroscedastic, two-factor nested designs with independent observations. These include Wald-type statistics based on the theory introduced by Akritas, Arnold and Brunner, as well as a Box-type approximation which is intended to improve the accuracy of approximation to asymptotic distributions. We also present statistics based on a recent theory of weighted F-statistics for ranks. The actual sizes of the statistics at various nominal levels are compared in a simulation study. Our main conclusion is that the Box-adjusted Wald-type statistic is the only statistic that is accurate across all the situations considered and therefore we recommend it for general use.

Keywords: ANOVA; Nested designs; Nonparametric hypotheses; Rank test statistics

1. Introduction

In this paper we shall study the application of non-parametric tests in the analysis of two-factor nested designs. In a *nested or hierarchical* design with the levels of factor B nested under the levels of factor A, each level of B occurs with only one level of A. A typical example is the following [10].

Four chemical companies produce three, two, two and four insecticide products, respectively. No two products are the same. Thirty-three boxes each containing 400 mosquitoes were allocated, three to each product, in an experiment and the number of mosquitoes surviving after 4 h in each box was counted. Factor A (companies) has four levels and factor B (insecticides) has 11 levels, nested inside A.

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A nested design is characterized by the lack of correspondence between the levels of B that belong to different levels of A. This differentiates it from a crossed design. If each company produced the same types of product, then the crossed design (two-way factorial) Company x Type would be used.

We will now state the conventional parametric model and analysis of a two-factor nested design. In the following sections we shall give existing non-parametric tests and develop a new one, and then go on to a simulation study to compare the performance of the alternative test statistics. Finally we will give our recommendations about what should be applied in practice.

Let Y_{ijk} $(i = 1, 2, \ldots, a;\ j = 1, 2, \ldots, c_i;\ k = 1, 2 \ldots, n_{ij})$ be a set of $N = \sum_{i=1}^{a} \sum_{j=1}^{c_i} n_{ij}$ independent random variables, where Y_{ijk} is the kth observation from cell (i, j). They satisfy the nested parametric model

$$Y_{ijk} = \mu + \alpha_i + \delta_{ij} + \epsilon_{ijk}, \tag{1}$$

where $\sum_{i=1}^{a} c_i\alpha_i = \sum_{j=1}^{c_i} w_{ij}\delta_{ij} = 0$ with $\sum_{j=1}^{c_i} w_{ij} = 1, \forall i$. We assume that the errors are independently and identically normally distributed with zero mean and fixed variance σ^2 (homoscedasticity). If the distribution of the errors is undefined, then (1) is a semiparametric linear model.

Consider testing the hypotheses $H_0(A)$ and $H_0(B|A)$ which correspond to the absence of A and B effects, respectively. In the parametric model (1), these are:

(1) $H_0(A) : \alpha_i = 0, \forall i = 1, \ldots, a$. The classical ANOVA test statistic is

$$F^{A}_{AOV} = \frac{(N - n_c) \sum\limits_{i=1}^{a} n_{i.}(\bar{Y}_{i..} - \bar{Y}_{...})^2}{(a-1) \sum\limits_{i=1}^{a} \sum\limits_{j=1}^{c_i} \sum\limits_{k=1}^{n_{ij}} (Y_{ijk} - \bar{Y}_{ij.})^2} \sim F_{a-1, N-n_c}$$

where $n_c = \sum_{i=1}^{a} c_i$, $\bar{Y}_{ij.} = n_{ij}^{-1} \sum_{k=1}^{n_{ij}} Y_{ijk}$, $\bar{Y}_{i..} = c_i^{-1} \sum_{j=1}^{c_i} \bar{Y}_{ij.}$ and $\bar{Y}_{...} = a^{-1} \sum_{i=1}^{a} \bar{Y}_{i..}$. This F-statistic is appropriate if we use the weights $w_{ij} = n_{ij}$. Otherwise the statistic has no closed form and we have to make use of a more general form, using projection matrices (Arnold [5]).

(2) $H_0(B|A) : \delta_{ij} = 0, \forall j = 1, \ldots, c_i, ; i = 1, \ldots, a$. The classical ANOVA test statistic is

$$F^{B}_{AOV} = \frac{(N - n_c) \sum\limits_{i=1}^{a} \sum\limits_{j=1}^{c_i} n_{ij}(\bar{Y}_{ij.} - \bar{Y}_{i..})^2}{(n_c - a) \sum\limits_{i=1}^{a} \sum\limits_{j=1}^{c_i} \sum\limits_{k=1}^{n_{ij}} (Y_{ijk} - \bar{Y}_{ij.})^2} \sim F_{n_c-a, N-n_c}.$$

2. Rank transform (RT) tests

From among the existing non-parametric versions of tests for the two hypotheses, we first mention briefly the rank transform (RT) statistics. These are based on the suggestion by Conover and Iman [7] that non-parametric tests for ANOVA can be obtained by ranking all the data values, regardless of which factor levels they belong to, and then substituting the ranks in place of the original values in the formula for the parametric F statistic. In practice, a statistic based on ranks is usually tested as a χ^2 divided by its degrees of freedom ([8] for the balanced two-way layout, [9] for randomized complete blocks). This procedure gives the crude RT statistic for $H_0(A) : \alpha_i = 0$,