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Linear contrasts for the one way analysis of variance: A Bayesian approach

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HIGHLIGHTS

- Linear contrasts between means are dealt with for the first time as objective Bayesian model selection problems.
- A specific solution for the homoscedastic case is proposed.
- The *p*-value and the posterior probability of the null hypothesis are compared through calibration curves.

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ABSTRACT

Linear contrasts between means for the one way analysis of variance are studied for the first time as objective model selection problems. For it, Bayes factors for intrinsic priors are used and classical and Bayesian measures of evidence are compared.

1. Introduction

Let us consider k normal populations $N(x_1|\mu_1, \sigma_1^2), \ldots, N(x_k|\mu_k, \sigma_k^2)$ and independent samples, $\mathbf{x}_i = (x_{i1}, \ldots, x_{in_i})$, from each population $i = 1, \ldots, k$. When the hypothesis of equality of means is rejected an analysis of certain linear contrasts between the means may be of interest. In the frequentist methodology there are several exact tests dealing with this topic, the methods of Scheffé and Tukey are the most commonly used in the homoscedastic case; however, just asymptotic solutions like the Welch's test or the Hotelling's test are obtained when heteroscedasticity is present. The first objective in this paper is to go one step further than in Cano et al. (2013), where the homoscedastic case for the one way ANOVA was dealt with using the intrinsic priors methodology. Here we solve as a model selection problem, linear contrasts like the following

$$H_0 : \varphi = 0 \quad \text{versus} \quad H_1 : \varphi \neq 0, \tag{1}$$

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where $\varphi = \sum_{i=1}^{k} a_i\mu_i$, with $\sum_{i=1}^{k} a_i = 0$ and at least one $a_i \neq 0$. This interesting classical problem is treated here for the first time as an objective Bayesian model selection one, note that in Box and Tiao (1973) it was dealt with from a Bayesian estimation point of view. In Cano et al. (2013) we argue why this type of problems are better treated from a Bayesian model selection perspective. For it, because Bayes factors for improper noninformative priors are undefined we propose Bayes factors based on the intrinsic methodology. See Berger and Pericchi (1996), Moreno et al. (1998), and Bertolino et al. (2000), where contrasts are briefly discussed as estimation problems. The case of large k is not dealt with as it is behind the scope of this paper.

The second objective is to compare the Bayesian measure of evidence, the posterior probability of the null hypothesis, y, with the frequentist one, the p-value, p. Calibration is a simple means of establishing that comparison, see Girón et al. (2006). We state in this paper that in linear contrasts the posterior probability of the null hypothesis depends on the sample through sufficient statistics and the sample size, and the same is true for the p-value. That is,

$$y = P(H_0|\bar{\mathbf{x}}, \mathbf{s}^2, \mathbf{n}), \tag{2}$$

$$p = P_{H_0}(T \geq t(\bar{\mathbf{x}}, \mathbf{s}^2, \mathbf{n})), \tag{3}$$

where T is the contrast and t is its observed value, $\bar{\mathbf{x}} = (\bar{x}_1, \dots, \bar{x}_k)$, $\mathbf{s}^2 = (s_1^2, \dots, s_k^2)$ and $s_i^2 = \sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2$; therefore we can define different calibration curves varying in (2) and (3) one of the sample means in an interval. Note that the posterior probability of the null hypothesis has been computed using the prior $p_1 = p_2 = 1/2$ for the hypotheses H_0 and H_1 and the p-values are the corresponding to the Scheffé test in the homoscedastic case and to the Welch test in the heteroscedastic one.

The paper is organized as follows. In Section 2 linear contrasts between means for homoscedastic populations are considered using the intrinsic priors methodology. In Cano et al. (2013) it is argued that it was necessary to study the case when homoscedasticity is present, since in this case a specific method can be used, similarly to what happens in the frequentist analysis.

In Section 3 Bayes factors for intrinsic priors are obtained for linear contrasts in the heteroscedastic case. The key idea to develop Sections 2 and 3 was to find a reparameterization allowing to formulate linear contrasts as nested Bayesian model selection problems for which the intrinsic methodology behaves satisfactorily, see Girón et al. (2006) and references therein. This provides us exact solutions even for the heteroscedastic case.

In Section 4 we illustrate the behavior of the calibration curves as the sample size of the involved populations increases. Finally, in Section 5 we briefly summarize the obtained results and we give some concluding remarks.

2. Linear contrasts between means for homoscedastic populations

In this section we consider k normal populations with unknown common variance σ^2 and we want to solve linear contrasts as (1) where, without loss of generality, we assume that $a_1 \neq 0$. The null hypotheses introduce a constraint on the parameters and considering the reparameterization

$$\varphi_1 = \sum_{i=1}^{k} a_i\mu_i, \varphi_2 = \mu_2, \dots, \varphi_k = \mu_k, \tag{4}$$

the linear contrast (1) can be expressed as a nested Bayesian model selection problem where the simple model M_1,

$$f_1(\mathbf{z}|\theta_1) = N_{n_1}\left(\mathbf{x}_1 \middle| \left(\sum_{i=2}^{k} d_i\beta_i\right)\mathbf{1}_{n_1}, \tau^2 I_{n_1}\right) \prod_{i=2}^{k} N_{n_i}(\mathbf{x}_i|\beta_i 1_{n_i}, \tau^2 I_{n_i}),$$

with the prior

$$\pi_1^N(\theta_1) = \frac{c_1}{\tau}, \tag{5}$$

is compared with the complex model M_2

$$f_2(\mathbf{z}|\theta_2) = \prod_{i=1}^{k} N_{n_i}\left(\mathbf{x}_i|\varphi_i 1_{n_i}, \sigma^2 I_{n_i}\right),$$

with the prior

$$\pi_2^N(\theta_2) = \frac{c_2}{\sigma}, \tag{6}$$

where $\mathbf{z} = (\mathbf{x}_1, \dots, \mathbf{x}_k)$, $\theta_1 = (\beta_2, \dots, \beta_k, \tau)$, $\theta_2 = (\varphi_1, \dots, \varphi_k, \sigma)$ and $d_i = -a_i/a_1$ for $i = 2, \dots, k$. Note that just $(k-1)$ means, $\beta_2, \dots, \beta_k$, have been left in model M_1 because of the constraint on the parameters.

To assign default priors we have always assumed that location and scale parameters are *a priori* independent, see Jeffreys (1961).