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Modelling circular random variables with a spike at zero

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ABSTRACT

This paper discusses models for circular responses with a spike at zero. Maximum likelihood estimation for the underlying parameters and a test for checking a spike are also carried out. Simulations and a real data example are considered for illustrations.

1. Introduction

Circular data is one less studied research topic in statistical literature, although this kind of data naturally occurs in some clinical trials and biomedical problems including ophthalmologic and orthopaedic studies. In the present paper, we first discuss a real data example in a clinical trial with circular responses which will be central to our present study.

We consider a study conducted at the Disha Eye Hospital and Research Center, Barrackpore, West Bengal, India, over a period of two years (2008–2010), and the data obtained from that study; see Bakshi (2010) for a detailed description of the study and data. A randomized unbiased allocation resulted 19 patients to undergo SICS with Snare technique (Basti et al., 1993), 18 patients to undergo SICS with Vectis technique (see Masket, 2004), 17 patients with Conventional Phacoemulsification (Zacharias, 2008) and 16 patients with Torsional Phacoemulsification (Mackool and Brint, 2004).

During the cataract surgery, incision causes unwanted changes to the natural corneal shape causing an astigmatic eye. It is preferred that the axis is closer to 0, 90 or 180°. To make only one preferred direction, we multiply by 4 (mod 360°). Consequently, the preferred angle reduces to 0° (=360°); the multimodal distribution becomes a distribution having a single mode at 0°.

Our preliminary analyses showed no significant treatment difference if we look at the angle of astigmatism only. Part of that analyses is provided in Biswas et al. (in press-a, in press-b). As we consider post-operative astigmatism only in the present paper, we consider all the observations are coming from a single distribution. The objective of our modelling is to estimate the success of the surgery with respect to the astigmatism in the patients after the operation. Interestingly, out of the 70 observations, 32 are zero, that is more than 45% of the data. The goodness of fit test using Watson's test (Watson, 1983) indicated good fit to the von Mises distributions for our data set excluding the 32 zero-values; the P-value being 0.0537. The expression for the probability density function (pdf) of the von Mises distribution is given in Section 3. So the combined

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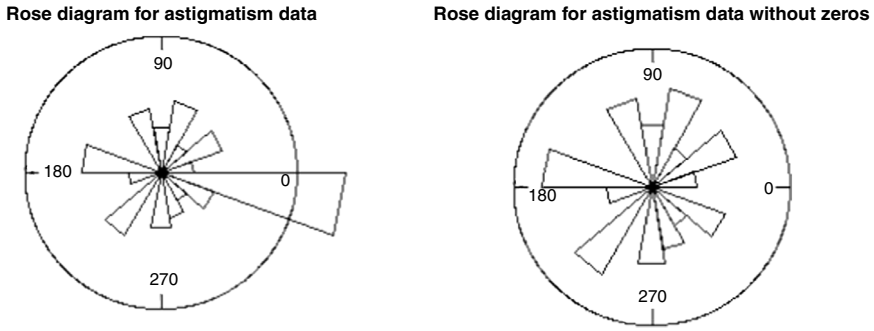

Fig. 1. Rose diagram of the data with and without zero-values.

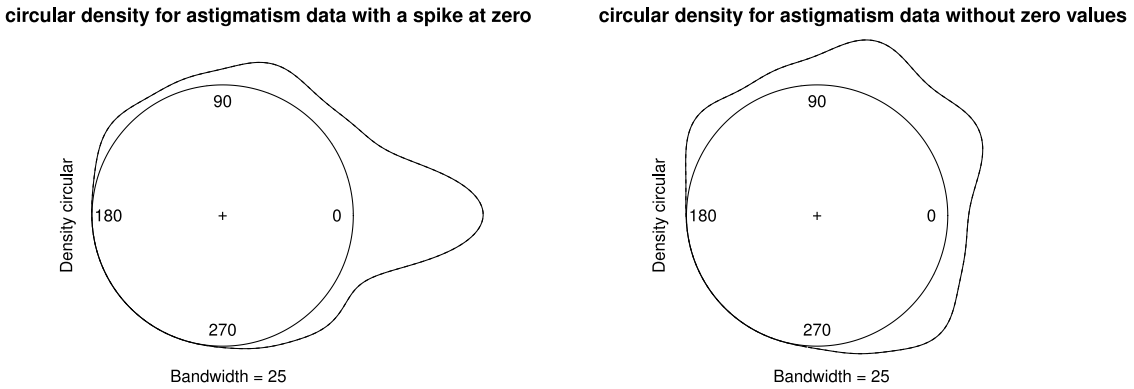

Fig. 2. Density plot of the data with and without zero-values.

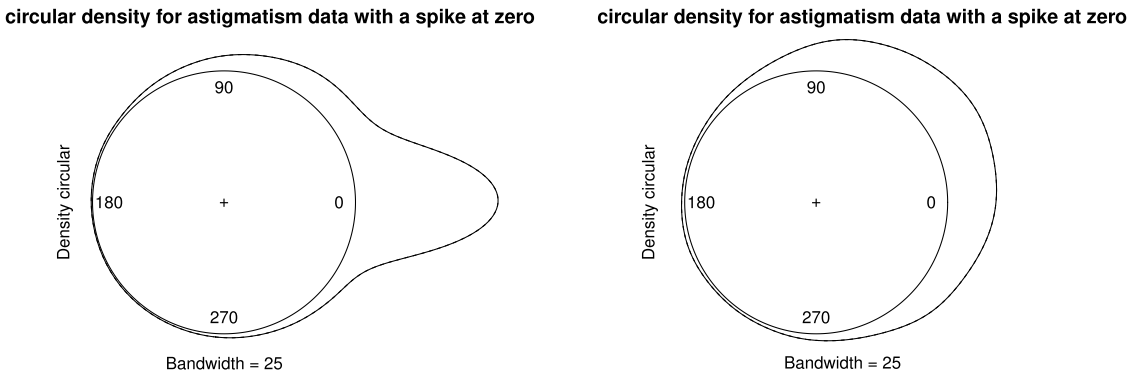

Fig. 3. Density plot of the VM($\mu = 25^0,\ \kappa = 1.5$). Left: mixed with 45% zeros; Right: no mixture with zeros.

data set may be considered to be from von Mises distribution with a spike at zero. We try to model the data accordingly in this paper. In fact, so far our knowledge goes, there is no work in the circular distribution with a spike (be it at zero or elsewhere).

The data set is presented in Rose diagram in Fig. 1, where an observation is the axis of astigmatism induced three months after the surgery. The left panel of Fig. 1 is for the full data, while the right panel is the rose diagram excluding the zero values. Note that by "data" we mean 4× observed axis of astigmatism (modulo 360°).

Also, the density estimate of the full data and the part of the data excluding zeros are shown in Fig. 2 where again the left panel shows the estimated density with full data and the right panel is the same for the data excluding the zero values. Also the left panel of Fig. 3 illustrates the density of a von Mises distribution with $\mu = 25^\circ$ and $\kappa = 1.5$ mixed with 45% zero values, while the right panel is the density of $VM(25^0, 1.5)$ without a mixture with zeros. Clearly the Figure mixed with zeros resembles the actual data plot better. In the present paper, we intend to fit the model mixed with zeros.

The rest of the paper is organized as follows. In Section 2, we discuss the circular normal with zero spike model. The estimation of the parameters is discussed in Section 3. We also carry out a test procedure to check whether the data is from