Statistics and Probability Letters 107 (2015) 27–36

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Stability in distribution of neutral stochastic functional differential equations☆

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ARTICLE INFO

Article history:
Received 22 October 2014
Received in revised form 14 June 2015
Accepted 25 July 2015
Available online 7 August 2015

MSC:
39B72
37H10

Keywords:
Stability in distribution
Neutral stochastic functional differential equations
Wiener processes
Jump processes

ABSTRACT

In this note, by the weak convergence approach, we investigate stability in distribution for a range of neutral stochastic functional differential equations. As a byproduct, our result derived also implies existence of invariant measure of the semigroup generated by the segment process. Last but not least, we close some gaps of Lemma 2, and improve greatly Lemma 4 due to Hu and Wang (2012).

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1. Introduction

Many physical phenomena have already been successfully modeled by stochastic dynamical systems whose evolution in time is governed by random forces as well as intrinsic dependence on the state over a finite interval of its past history. Such models may be identified as functional stochastic differential equations (SDEs) (see, e.g., Bao et al., 2014a). In response to the great needs, there is an extensive literature on functional SDEs, see, e.g., the monographs due to Mohammed (1984), Mao (2008), Mao and Yuan (2006), respectively. For finite-time/long-time behavior of functional SDEs, we refer to, e.g., Bao and Yuan (2015) on large deviation principle, Bao and Yuan (2014a) on numerical approximation, Bao et al. (2014b) for ergodicity.

As far as stochastic dynamical systems are concerned, stability in probability or in mean or in almost sure sense is a little bit strong compared with stability in distribution. The pioneer work on stability in distribution is due to Basak and Bhattacharya (1992), where a class of singular diffusions were discussed. Since then, the subject on stability in distribution has been investigated in various setup, see, e.g., Basak et al. (1996) for semi-linear SDEs with Markovian switching, Yuan and Mao (2003) for nonlinear SDEs with Markovian switching, Yuan et al. (2003) for functional SDEs, Tan et al. (2013) for

☆ Supported in part by NNSF (No. 11301230, 71371193) and Project funded by Jiangxi University of Finance and Economics.

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http://dx.doi.org/10.1016/j.spl.2015.07.033
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functional SDEs with variable delays. Moreover, with regard to stability in distribution of numerical schemes, we refer to, e.g., Yuan and Mao (2004, 2005), Mao et al. (2005), and Bao and Yuan (2014b).

An SDE is called a *neutral* functional SDE if it not only depends on the past and the present values but also involves derivatives with delays, see, e.g., Chapter 6 of Mao (2008). This kind of equation has also been utilized to model some evolution phenomena arising in physics, biology and engineering, see, e.g., Kolmanovskii and Nosov (1981) for the theory in aeroelasticity, Mao (2008) for the collision problem in electrodynamics, Slemrod (1971) for the oscillatory systems. Generally, a neutral SDE admits the following form

$$\mathrm{d}\{X(t)-u(X_t)\}=f(X_t)\mathrm{d}t+g(X_t)\mathrm{d}B(t),\quad t>0,\qquad X_0=\xi. \tag{1.1}$$

Here, u is refered to as a *neutral term* and $\{B(t)\}_{t\geq 0}$ is standard Brownian motion.

As for stability in distribution for neutral functional SDEs with point delay, we refer to, e.g., Bao et al. (2009). Hu and Wang (2012) generalized Bao et al. (2009) to the case of distributed delay. However, there are some big gaps in the proof of Lemmas 2 and 4 therein, which, whereas, play a crucial role on the long term behavior analysis for the segment process. In this note, we shall reconsider stability in distribution for neutral functional SDEs driven by Brownian motions or jump processes, and put forward some tricks to refine the arguments of Lemmas 2 and 4 in Hu and Wang (2012). Moreover, our result also improves the corresponding one of Bao et al. (2009) to the distributed case.

The paper is organized as the following: Section 2 focuses on stability in distribution of neutral functional SDEs driven by Brownian motions; Section 3 is devoted to stability in distribution of neutral functional SDEs with pure jumps. To illustrate the theory established, an illustrative example is provided in the last section.

2. Stability in distribution: Brownian motion case

For each integer $n>0$, let $(\mathbb{R}^n,\langle\cdot,\cdot\rangle,|\cdot|)$ be an n-dimensional Euclidean space. Denote $\mathbb{R}^n\otimes\mathbb{R}^m$ by the set of all $n\times m$ matrices A endowed with Hilbert–Schmidt norm $\|A\|:=\sqrt{\mathrm{trace}(A^TA)}$, in which A^T is the transpose of A. For fixed $\tau\in(0,\infty)$, which will be referred to as the delay or memory, let $\mathscr{C}=C([-\tau,0];\mathbb{R}^n)$ be continuous functions $\xi:[-\tau,0]\mapsto\mathbb{R}^n$ equipped with the uniform norm $\|\zeta\|_\infty:=\sup_{-\tau\leq\theta\leq 0}|\zeta(\theta)|$. $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq0},\mathbb{P})$ is a filtered complete probability space, where $\mathscr{F}$ is a σ-algebra on the outcome space Ω, $\mathbb{P}$ is a probability measure on the measurable space $(\Omega,\mathscr{F})$, and $\{\mathscr{F}_t\}_{t\geq0}$ is a filtration of sub-σ-algebra of $\mathscr{F}$, where the usual conditions are satisfied, i.e., $(\Omega,\mathscr{F},\mathbb{P})$ is a complete probability space, and for each $t\geq0$, $\mathscr{F}_t$ contains all $\mathbb{P}$-null sets of $\mathscr{F}$ and $\mathscr{F}_{t_+}:=\bigcap_{s>t}\mathscr{F}_s=\mathscr{F}_t$. Let $\{B(t)\}_{t\geq0}$ be an m-dimensional Brownian motion defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq0},\mathbb{P})$. If $X(t)$ is a continuous $\mathbb{R}^n$-valued stochastic process on $t\in[-\tau,\infty)$, we let $X_t(\theta)=\{X(t+\theta),-\tau\leq\theta\leq0\}$ for all $t\geq0$, which is regarded as a $\mathscr{C}$-valued stochastic process.

For (1.1), we suppose that $f:\mathscr{C}\mapsto\mathbb{R}^n$, $g:\mathscr{C}\mapsto\mathbb{R}^n\otimes\mathbb{R}^m$, and $u:\mathscr{C}\mapsto\mathbb{R}^n$ are all measurable, locally bounded and continuous. Furthermore, we assume that

(A1) There exist $\lambda_1>\lambda_2>0,\ \lambda_3>0,\ \kappa\in(0,1)$ and a probability measure $\rho(\cdot)$ on $[-\tau,0]$ such that for all $\phi,\ \varphi\in\mathscr{C}$

$$\begin{aligned}&2\langle(\phi(0)-\varphi(0))-(u(\phi)-u(\varphi)),f(\phi)-f(\varphi)\rangle+\|g(\phi)-g(\varphi)\|^2\\&\quad\leq-\lambda_1|\phi(0)-\varphi(0)|^2+\lambda_2\int_{-\tau}^0|\phi(\theta)-\varphi(\theta)|^2\rho(\mathrm{d}\theta);\end{aligned} \tag{2.1}$$

$$\|g(\phi)-g(\varphi)\|^2\leq\lambda_3\left(|\phi(0)-\varphi(0)|^2+\int_{-\tau}^0|\phi(\theta)-\varphi(\theta)|^2\rho(\mathrm{d}\theta)\right); \tag{2.2}$$

$$|u(\phi)-u(\varphi)|^2\leq\kappa\int_{-\tau}^0|\phi(\theta)-\varphi(\theta)|^2\rho(\mathrm{d}\theta). \tag{2.3}$$

Remark 2.1. As to (1.1), the existence and uniqueness of the solution process can be guaranteed by the following weaker condition compared with (2.1) (see, e.g., Bao et al., 2014a):

$$2\langle(\phi(0)-u(\phi)),f(\phi)\rangle+\|g(\phi)\|^2\leq\lambda\left(1+\|\phi\|_\infty^2\right)$$

for all $\phi\in\mathscr{C}$ and some $\lambda>0$. However, (2.1), which will be used to analyze stability in distribution for the segment process $X_t(\xi)$, guarantees that (1.1) admits a unique strong solution $\{X(t;\xi)\}_{t\geq-\tau}$ with the initial data ξ.

We should point that $X(t;\xi)\in\mathbb{R}^n$ is a point, whereas $X_t(\xi)\in\mathscr{C}$ is a continuous function on the interval $[-\tau,0]$ taking values in $\mathbb{R}^n$. Moreover, $\{X_t(\xi)\}_{t\geq0}$ admits a strong Markovian property according to Mohammed (1984). For $t\geq0$, denote by $P(\xi;t,\cdot)$ the transition probability of $X_t(\xi)$. Furthermore, denote $\mathscr{P}(\mathscr{C})$ the family of all probability measures on $(\mathscr{C},\mathscr{B}(\mathscr{C}))$. For any $P_1,P_2\in\mathscr{P}(\mathscr{C})$, define the metric $\mathrm{d}_{\mathbb{L}}$ by

$$\mathrm{d}_{\mathbb{L}}(P_1,P_2)=\sup_{f\in\mathbb{L}}\left|\int_{\mathscr{C}}f(\xi)P_1(\mathrm{d}\xi)-\int_{\mathscr{C}}f(\eta)P_2(\mathrm{d}\eta)\right|,$$

where

$$\mathbb{L}=\{f:\mathscr{C}\to\mathbb{R}:|f(\xi)-f(\eta)|\leq\|\xi-\eta\|_\infty\text{ and }|f(\cdot)|\leq1\text{ for }\xi,\eta\in\mathscr{C}\}.$$

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