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A generalization of quantile-based skew logistic distribution of van Staden and King

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ABSTRACT

We present a generalization of the quantile-based skew logistic distribution of van Staden and King (2015), providing a wider range of *L*-skewness and *L*-kurtosis. We also discuss the estimation of model parameters by the method of *L*-moments and derive the asymptotic distribution.

1. Introduction

The logistic distribution is a commonly used model in health sciences. It is a simple continuous symmetric distribution with many attractive properties including an increasing hazard rate. For an elaborate discussion on the theory, methods and applications of logistic distribution, interested readers may refer to Johnson et al. (1995) and Balakrishnan (1992).

Recently, van Staden and King (2015) used a quantile-based approach to define a skew logistic distribution, which was originally introduced by Gilchrist (2000), and is denoted here by SLD_{QB}; see also Nair et al. (2013). van Staden and King (2015) extended his work by investigating the properties of SLD_{QB} and providing closed-form estimators for the parameters. First, the quantile function of the standard logistic distribution is

$$Q_0(p) = \log(p) - \log(1-p), \quad 0 < p < 1.$$

These authors then proposed the SLD_{QB} as one with its quantile function as

$$Q_{SLD}(p) = \alpha + \beta\left((1-\delta)\log(p) - \delta\log(1-p)\right), \tag{1}$$

where $0 < p < 1, 0 < \delta < 1, \beta > 0$ and $\alpha \in \mathbb{R}$. The *L*-skewness is limited to be in the interval $(-\frac{1}{3}, \frac{1}{3})$ while the *L*-kurtosis is fixed at $\frac{1}{6}$. In the present work, we provide a generalization of this model and discuss its properties and a method of estimation of its parameters.

The rest of this paper is organized as follows. In Section 2, we provide the definition of the generalized quantile-based skew logistic model ($GSLD_{QB}$) and discuss its properties. In Section 3, we present a method of estimation for the model

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parameters through the method of L-moments; see Hosking (1990). Asymptotic distribution of the estimators is also presented along with explicit expressions for the asymptotic covariance matrix of the estimators. Finally, an example is provided in Section 4 to illustrate the model and the inferential results developed here.

2. Generalized skew logistic model

2.1. Definition

In the model provided by van Staden and King (2015), the skew form of the standard logistic distribution is obtained by a weighted sum of quantile functions of standard exponential distribution, $Q_e(p) = -\log(1-p)$, and of standard reflected exponential distribution, $Q_{re}(p) = \log(p)$, namely,

$$Q_{SLD,0}(p) = (1-\delta)\log(p) - \delta\log(1-p), \quad 0 < p < 1, 0 \le \delta \le 1.$$

Though this model provides flexibility in skewness through parameter δ, the L-kurtosis remains constant at $\frac{1}{6}$. So, for the purpose of providing more flexibility to this distribution, we propose a more general quantile function of the form

$$Q_{GSLD,0}(p) = (1-\delta)\log(p^{\kappa}) - \delta\log(1-p^{\kappa}), \tag{2}$$

where $0 < p < 1, 0 \le \delta \le 1$ and $\kappa > 0$. The power parameter κ is intended to provide a range of possible L-kurtosis values. The following generalized definition incorporates location and scale parameters as well.

Definition 1. A real-valued random variable X is said to have a generalized quantile-based skew logistic distribution, denoted by $X \sim GSLD_{QB}(\alpha, \beta, \delta, \kappa)$, if its quantile function is given by

$$Q_{GSLD}(p) = \alpha + \beta\{(1-\delta)\log(p^{\kappa}) - \delta\log(1-p^{\kappa})\}, \tag{3}$$

where α and β (> 0) are location and scale parameters, respectively, and δ $(0 \le \delta \le 1)$ and κ (> 0) are shape parameters.

A reflective definition can be simply done by

$$Q_{GSLD,reflected}(p) = -Q_{GSLD}(1-p).$$

Since the properties of this distribution will be similar to that of $GSLD_{QB}$, we focus only on $GSLD_{QB}$ in the rest of the paper.

The parameters δ and κ both control the level of skewness. When $\kappa = 1$, the $GSLD_{QB}$ will be reduced to SLD_{QB} of van Staden and King (2015), and this model is negatively (positively) skewed if $\delta < \frac{1}{2}$ $(\delta > \frac{1}{2})$. When $\kappa \to 0$, $GSLD_{QB}$ is positively skewed. When κ increases, the skewness decreases if $\delta < \frac{1}{2}$, while it increases and then decreases if $\frac{1}{2} < \delta < 1$.

2.2. Quantile density and moment-generating functions

Theorem 1. *The quantile density function of $GSLD_{QB}$ is*

$$q(p) = \beta\left(\frac{\kappa(1-\delta)}{p} + \frac{\delta\kappa p^{\kappa-1}}{1-p^{\kappa}}\right), \tag{4}$$

its density quantile function is

$$f_p(p) = \frac{p(1-p^{\kappa})}{\beta\kappa(1-\delta-p^{\kappa}+2\delta p^{\kappa})}, \tag{5}$$

and its moment generating function (MGF) is

$$\begin{aligned} M(t) &= E(\exp(Xt)) = \int_0^1 \exp(Q(p)t)dp \\ &= \int_0^1 \exp(\alpha t + \beta t((1-\delta)\log(p^{\kappa}) - \delta\log(1-p^{\kappa})))dp \\ &= k^{-1}\exp(\alpha t)B(t\beta(1-\delta) + 1/k, 1-\beta\delta t), \end{aligned} \tag{6}$$

where $B(a, b)$ denotes the complete beta function.

2.3. Moments

The moments can be readily obtained from the derivatives of the moment generating function in (6). For example, the rth moment can be obtained as $E(X^r) = M^{(r)}(0) = \left.\frac{d^r M(t)}{dt^r}\right|_{t=0}$. In the following theorem, we present expressions for the mean, variance and coefficients of skewness and kurtosis.